The Laplace transform of a function $f(t)$ is the function $F(s) := \mathcal{L}\{f(t)\}(s) = \int_0^\infty f(t)e^{-st}dt$.

1. Compute the Laplace transform of the function $f(t) = e^{at}$. What is the domain of $F(s)$?

2. Let $f(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & t > 1 \end{cases}$. Sketch a graph of $f(t)$ and compute its Laplace transform.
3. If $f(t) = t$, compute $\mathcal{L}(f)$.

4. Let $f(t) = \begin{cases} 
  t & 0 \leq t \leq 1 \\
  1 & t > 1
\end{cases}$ Sketch a graph of $f(t)$ and compute its Laplace transform.