MthSc 208: Differential Equations (Spring 2011)
In-class Worksheet 5f: ODEs with Piecewise Forcing Terms

NAME:

Consider the initial value problem $y'' + y = f(t)$, $y(0) = 0$, $y'(0) = 1$, where $f(t) = \begin{cases} 2t, & 0 \le t \le 1 \\ 2, & t > 1 \end{cases}$

1. Sketch $f(t)$, and write it using the Heavyside function.

2. Take the Laplace transform of the differential equation, and solve for $Y(s)$.

3. Use partial fractions to decompose $Y(s)$ into four terms. [Note: $\dfrac{1}{s^2(s^2+1)} = \dfrac{1}{s^2} - \dfrac{1}{s^2+1}$.]

4. Apply the inverse Laplace transfrom to each term and write the solution to the IVP using the Heavyside function.

5. Write the solution as a piecewise function (i.e., *not* using the Heavyside function).