1. Let $T\colon X \to U$ be a linear map. Prove the following:
   (a) The image of a subspace of X is a subspace of U.
   (b) The inverse image of a subspace of U is a subspace of X.

2. Prove Theorem 3.3 in Lax:
   (a) The composite of linear mappings is also a linear mapping.
   (b) Composition is distributive with respect to the addition of linear maps, that is,

   $$(R+S)\circ T = R\circ T + S\circ T$$

   and

   $$S\circ(T+P) = S\circ T + S\circ P,,$$

   where $R, S\colon U \to V$ and $P, T\colon X \to U$.

3. Suppose that X, U, and V are vector spaces, and $S\colon X \to U$ and $T\colon X \to V$ are linear maps, and T is surjective. Give necessary and sufficient conditions for the existence of a well-defined linear map $R\colon U \to V$ such that $RS = T$. Prove all of your claims.

4. Let X be a finite-dimensional vector space over K and let $\{x_1, \dots, x_n\}$ be an ordered basis for X. Let U be a vector space over the same field K but possibly with a different dimension, and let $\{u_1, \dots, u_n\}$ be an arbitrary set of vectors in U. Show that there is precisely one linear transformation $T\colon X \to U$ such that $Tx_i = u_i$ for each $i = 1, \dots, n$.