1. Show that whenever meaningful,

   (i) $(ST)' = T'S'$

   (ii) $(T + R)' = T' + R'$

   (iii) $(T^{-1})' = (T')^{-1}$.

   Here, S' denotes the transpose of S. Carefully describe what you mean by "whenever meaningful" in each case.

2. Give a direct algebraic proof of $N_{T'}^{\perp} = R_T$ (i.e., don't just use the fact that $N_{T'} = R_T^{\perp}$ and take the annihilator of both sides.)

3. Let X be a finite-dimensional vector space and $A, B \in \mathscr{L}(X, X)$ (linear functions $X \to X$).

   (a) Show that if A is invertible and similar to B, then B is also invertible, and B^{-1} is similar to A^{-1}.

   (b) Show that if either A or B is invertible, then AB and BA are similar.

4. Suppose $T\colon X \to X$ is a linear map of rank 1, and $\dim X < \infty$.

   (a) Show that there exists a unique $c \in K$ such that $T^2 = cT$.

   (b) Show that if $c \neq 1$, then $I - T$ has an inverse.

5. Suppose that $S, T\colon X \to X$ and $\dim X < \infty$.

   (a) Show that $\operatorname{rank}(S + T) \leq \operatorname{rank}(S) + \operatorname{rank}(T)$.

   (a) Show that $\operatorname{rank}(ST) \leq \operatorname{rank}(S)$.

   (b) Show that $\dim(N_{ST}) \leq \dim N_S + \dim N_T$.