1. Consider the square wave defined by the function $f(t)$, where

$$f(t) = \begin{cases} 1 & 0 \le t < 1, \\ -1 & -1 \le t < 0, \end{cases}$$

and f is extended to be periodic of period $T = 2$.

(a) Sketch the graph of $f(t)$ on $[-7, 7]$.

(b) Compute the Fourier series of $f(t)$.

(c) The differential equation

$$x''(t) + \omega_0^2 x(t) = f(t)$$

describes the motion of a simple harmonic oscillator, subject to a driving force given by the square wave $f(t)$. Find the general solution by first solving the homogeneous equation, and then looking for a particular solution of the form

$$x_p(t) = \sum_{n=1}^{\infty} b_n \sin(n\pi t)\,.$$

2. Consider the 2π-periodic function defined on $[-\pi, \pi]$ by

$$f(t) = \begin{cases} 0 & -\pi \le t < 0, \\ t & 0 \le t \le \pi, \end{cases}$$

(a) Sketch the graph of $f(t)$ on $[-7\pi, 7\pi]$.

(b) Compute the Fourier series of $f(t)$.

(c) Sketch the graph of the resulting Fourier series. [It will be the same as the answer to Part (a) *except* at the points of discontinuity.]

(d) Solve the differential equation $x''(t) + \omega_0^2 x(t) = f(t)$. Look for a particular solution of the form

$$x_p(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt\,.$$

3. Determine which of the following functions are even, which are odd, and which are neither even nor odd.

(a) $f(x) = x^3 + 3x$	(e) $f(x) = \sin 2x$
(b) $f(x) = x^2 + \|x\|$	(f) $f(x) = e^x$
(c) $f(x) = \frac{1}{x}$	(g) $f(x) = \frac{1}{2}(e^x + e^{-x})$
(d) $f(x) = x \cos x$	(h) $f(x) = \frac{1}{2}(e^x - e^{-x})$.

4. Consider the $2L$-periodic function $f(x) = \dfrac{a_0}{2} + \displaystyle\sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}$. Write the Fourier series for the following functions:

(a) The reflection of $f(x)$ across the y-axis;

(b) The reflection of $f(x)$ across the x-axis;

(c) The reflection of $f(x)$ across the origin.

5. (a) The Fourier series of an odd function consists only of sine-terms. What additional symmetry conditions on f will imply that the sine coefficients with even indices will be zero (i.e., each $b_{2n} = 0$)? Give an example of a non-zero function satisfying this additional condition.

   (b) What symmetry conditions on f will imply that the sine coefficients with odd indices will be zero (i.e., each $b_{2n+1} = 0$)? Give an example of a non-zero function satisfying this additional condition.

   (c) Sketch the graph of a non-zero even function, such that $a_{2n} = 0$ for all n.

   (d) Sketch the graph of a non-zero even function, such that $a_{2n+1} = 0$ for all n.

6. Consider the function $f(x) = x(L - x)$ defined on the interval $[0, L]$. For this problem, you will determine the Fourier series, Fourier cosine series, and Fourier sine series of $f(x)$. Feel free to use a computer to find any indefinite integrals that you need.

   (a) Sketch the even extension of f and compute its Fourier cosine series.

   (b) Sketch the odd extension of f and compute its Fourier sine series.

   (c) Sketch the periodic extension of f and compute its Fourier series.