Read: Stahl, Chapters 1.1, 1.2, 1.3, 1.4, 2.1.

1. Let X be a geometry, and let f and g be isometries of X.

   (a) Prove that f is invertible and f^{-1} is an isometry of X.

   (b) Prove that $f \circ g$ is an isometry of X.

   For problems 2–6*, you are only allowed to use our analytic definition of Euclidean geometry and the results that we have derived from it.*

2. Prove that Euclidean length of a path is invariant under orientation-reversing change of parameter (recall that we did the orientation-preserving case in class).

3. Prove that the Euclidean angle between two paths at a given point of intersection is well-defined, that is, it is invariant under orientation-preserving change of parameter.

4. Prove that the translation $\tau_{\mathbf{v}}$ is a Euclidean isometry that preserves lines.

5. Prove that any two points in $\mathbf{E}^2$ can be moved to two points on the x-axis using a sequence of rotations around the origin and translations.

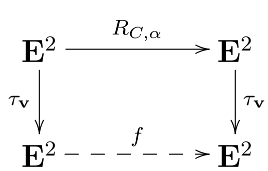
6. Prove that any pair of lines that meet at an angle θ can be moved by a Euclidean isometry onto any other pair of lines that meet at an angle θ.

   For problems 7–8*, you may use both analytic and ruler-and-compass methods.*

7. Prove that if two rigid motions agree on two distinct points, then they agree everwhere on the line joining those two points.

8. Let $\tau_{\mathbf{v}}$ be a translation, and let $R_{C,\alpha}$ be a rotation around a point $C \in \mathbf{E}^2$.

   (a) Describe, geometrically, the isometry $f := \tau_{\mathbf{v}} \circ R_{C,\alpha} \circ \tau_{\mathbf{v}}^{-1}$. This is the unique map that makes the following diagram commute:



   What do $R_{C,\alpha}$ and $\tau_{\mathbf{v}} \circ R_{C,\alpha} \circ \tau_{\mathbf{v}}^{-1}$ have in common?

   (b) Express $R_{C,\alpha}$ as a composition of translations and rotations around the origin.