Read: Stahl, Chapters 4.4., 5.1, 5.2, 5.3, 5.4.

1. Find the Euclidean center and radius of the circle that has hyperbolic center $(5, 4)$ and radius 3.

2. Prove that the hyperbolic circumference of a circle with hyperbolic radius R is $2\pi \sinh(R)$.

3. Prove Proposition 5.1.4 in Stahl, that in hyperbolic geometry, every angle is congruent to an angle in standard position. Recall that in class we showed this for right angles, which was the hyperbolic analog of Euclid's Postulate 4.

4. Let P be a point in $\mathbf{H}^2$, and let ℓ be a Euclidean line segment through P. Prove that there exists a unique hyperbolic geodesic g that passes through P and is tangent to ℓ. (In other words, prove that, given a point P and a unit vector $\mathbf{v}$, there is a unique geodesic through P that leaves P in the direction of $\mathbf{v}$.) Also, give an explicit construction/description of g, given ℓ, using either ruler-and-compass or analytic methods.

5. Exercise 5.2.6 in Stahl asks you to consider the hyperbolic analog of the Euclidean theorem which states that the tangent line to a circle is perpendicular to the radius through the point of contact. Specifically, determine the hyperbolic analogs of the Euclidean terms in the statement of the Euclidean theorem (e.g., hyperbolic tangent line to a hyperbolic circle, radius of a hyperbolic circle), and either prove that the analogouus statement is true, or modifiy it as little as possible to make it true.

6. Suppose g and h are two geodesics in $\mathbf{H}^2$, which, as figures of the Euclidean plane, are tangent to each other.
   (a) Prove that the point of tangency is on the x-axis.
   (b) Prove that the two geodesics are hyperbolically asymptotic near that point of tangency.

Problems 7–8 *outline proofs of the lemmas needed for the "three reflections theorem" of the hyperbolic plane.*

7. (a) Let A and B be distinct points in $\mathbf{H}^2$. Prove that the geodesic segment from A to B has a unique (geodesic) perpendicular bisector.
   (b) Let A and B be distinct points in $\mathbf{H}^2$. Prove that there is a hyperbolic reflection ρ such that $\rho(A) = B$.

8. (a) Let A, B, and C be distinct points in $\mathbf{H}^2$. Prove that the (geodesic) angle bisector of $\angle BAC$ exists and is unique.
   (b) Let A, B, and C be distinct points in $\mathbf{H}^2$ such that $h(A, B) = h(A, C)$. Prove that there is a hyperbolic reflection ρ such that $\rho(B) = C$ and $\rho(A) = A$.