Modeling (overview)

Models are everywhere!

Physics

Chemistry

Engineering

Economics

Biology

Psychology

Sociology

Statistics

Meteorology

What is a model?

Ex: Physics Ball drops from a roof.

\[ x'' = -9.8, \quad x' = -9.8t, \quad x = -4.9t^2 \]

Finance Rate of an investment grows at a rate proportional to its quantity (e.g., \( r = 5\% \)).

E.g. \( P' = 0.05P \) \( \rightarrow \) \( P(t) = P_0 e^{0.05t} \)

Biology A colony of bacteria grows at a rate proportional to its size.

E.g. \( M' = kM \) \( \rightarrow \) \( M(t) = M_0 e^{kt} \)
Chemistry: A radioactive substance decays at a rate proportional to how much is remaining.

\[ M(t) = M_0 e^{-kt} \]

Physics: The temp of a cup of coffee cools at a rate proportional to 
\[(\text{temp of coffee}) - \text{(ambient temp, } A)\]

\[ T(t) = A + Ce^{-kt} \]

Goals with these models

- Analyze them (initial value? limiting value)
- Validate them (does the math reflect the actual phenomenon?)
More complicated models:

Falling object with air resistance: $F = ma = -mg$

with $w$: $F = -mg + R(v)$

What is $R(v)$? Try $R(v) = -rv$ (good approx.)

$$F = ma = mv' = -mg - rv \Rightarrow v' = -g - \frac{r}{m} v$$

Solve: $v(t) = -\frac{mg}{r} + C e^{-\frac{rt}{m}}$

Note: Initial vel: $v(0) = -\frac{mg}{r} + C$

Terminal vel: $\lim_{t \to \infty} v(t) = -\frac{mg}{r}$

Population growth

Logistic eqn: $y'(t) = r(y) \cdot y(t)$

$r(y)$ should be decreasing.

$y' = y(r - ay) = ry(1 - \frac{y}{M})$

Solve: $y(t) = \frac{M}{1 + Ce^{-rt}}$

Initial pop: $y(0) = \frac{M}{1 + C}$

Limiting pop: $\lim_{t \to \infty} y(t) = M$
Two "steady-states":
\[ y(t) = 0, \ M. \]
\[ y' > 0 \]
\[ y' < 0 \]

Threshold eq'n:

\[ y' = -r y \left( 1 - \frac{y}{M} \right) \left( 1 - \frac{y}{T} \right) \]

This modeled the passenger pigeon quite well!

Note: There is a lot of ecology/biology behind the scene:

- Gestation period
- \# of offspring
- Geography
- reproduction (sexual vs. asexual)