Example: Each day, a population reproduces.

It has: "growth rate" of $f$ 

"death rate" of $d$

\[ \Delta P = fP - dP = (f-d)P \]

let $P_t = P(t) = \text{population at time } t$.

Then \[ \Delta P = P_{t+1} - P_t \]

\[ \Rightarrow P_{t+1} = P_t + \Delta P = P_t + (f-d)P_t = (1+f-d)P_t \]

or just \[ P_{t+1} = \lambda P_t \]

Ex: $P_0 = 300$, $f = .03$, $d = .01$, $1+f-d = 1.02$

$P_1 = (1.02)P_0$

$P_2 = (1.02)P_1 = (1.02)^2P_0$

$P_3 = (1.02)P_2 = (1.02)^2P_0$

\[ \vdots \]
What is a difference eqn?

Formally: let $Q$ be a quantity defined for all $t \in \mathbb{N}$ such that $Q_{t+1} = F(Q_t)$.

Previous ex: $F(x) = \lambda x$ "Malthusian model"

^ this is linear.

Goal: Find a good model

Analyze models that arise.

Difference eqns are "discrete time, continuous space"

(compare to differential eqns: "continuous time & space")

Logistic difference eqn (one version)

Growth rate depends on size! (density dependent)

Death rate? Assume density independent. (why?)

Idea: Analyze $\frac{dP}{dt} = \text{per capita growth rate}$.
\[ \frac{\Delta P}{P} = -\frac{r}{M} P + r = r(1 - \frac{P}{M}) \]

or \[ P_{t+1} = P_t \left( 1 + r \left( 1 - \frac{P_t}{M} \right) \right) \]

"discrete logistic model."

Note:
- \( P \ll K \implies 1 - \frac{P}{K} \approx 1 \implies P_{t+1} \approx (1 + r) P_t \)
- \( P \approx K \implies P_{t+1} \approx 0 \)

Question: What is \( F(x) \)? (in class exercise.)

Remark: Though difference eqns are simple, they often have no closed form solution for \( P_t \)!

We can plot the solutions: 

\[ \text{pop. } P \]

\[ \text{time} \]
Cobwebbing: Suppose $p_{t+1} = F(p_t)$.

We can numerically find $P_0, P_1, P_2, \ldots$ as follows.

Think: When will the first one (toggling) be more realistic? "\ldots \ 2nd \ldots" be more realistic?