MthSc 311: Linear Algebra (Fall 2012)
Midterm 1
October 5, 2012

NAME:

Instructions

- Exam time is 50 minutes
- You may *not* use notes or books.
- Calculators are *not* allowed.
- **Show your work.** Partial credit will be given.

Question	Points Earned	Maximum Points
1		6
2		6
3		10
4		18
5		10
Total		**50**

Student to your left:

Student to your right:

1. Compute the inverse of the following matrix:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 5 & 6 \\ -2 & -7 & -9 \end{bmatrix}$$

2. Suppose you want to perform elimination on the following matrix

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 4 & -1 \\ 1 & 3 & 2 & -2 \\ 3 & 9 & 6 & -6 \end{bmatrix}$$

As a first step, you want to swap rows 1 and 2, and also subtract 3 times row 3 from row 4. Let $\boldsymbol{E}$ be the matrix that does this. What is $\boldsymbol{E}$, and do you want to compute $\boldsymbol{EA}$ or $\boldsymbol{AE}$? What would happen to $\boldsymbol{A}$ if you multiplied $\boldsymbol{E}$ on the "wrong side" of it? (You can either write down the explicit matrix or just describe it in words.)

3. For each statement below, circle the option that makes it correct.

   (a) Suppose $\boldsymbol{v}_1, \dots, \boldsymbol{v}_6$ are six vectors in $\boldsymbol{R}^4$.

      i. Those vectors (do)(do not)(might not) span $\boldsymbol{R}^4$.

      ii. Those vectors (are)(are not)(might be) linearly independent.

      iii. Any four of those vectors (are)(are not)(might be) a basis for $\boldsymbol{R}^4$.

   (b) Suppose $\boldsymbol{v}_1, \dots, \boldsymbol{v}_4$ are four vectors in $\boldsymbol{R}^6$.

      i. Those vectors (do)(do not)(night not) span a subspace of $\boldsymbol{R}^6$.

      ii. Those vectors (are)(are not)(might be) linearly independent.

      iii. Those four vectors (are)(are not)(might be) a basis for a subspace of $\boldsymbol{R}^6$.

   (c) The column vectors of an $n \times n$ matrix (do)(do not)(might not) span the column space.

   (d) The column vectors of an $n \times n$ matrix (are)(are not)(might be) a basis for the column space.

   (e) The column vectors of an $n \times n$ invertible matrix (do)(do not)(might) span $\boldsymbol{R}^n$.

   (f) The column vectors of an $n \times n$ invertible matrix (are)(are not)(might be) a basis for $\boldsymbol{R}^n$.

4. Consider the 3×4 system $\boldsymbol{Ax} = \boldsymbol{b}$, where $\boldsymbol{A} = \begin{bmatrix} 1 & 1 & -2 & 1 \\ -1 & -1 & 4 & 3 \\ 0 & 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$.

   (a) Use elimination to reduce $\boldsymbol{A}$ down to its *reduced-row echelon form.* That is, all pivots should be 1, and with zeros below *and* above them.

   (b) Describe in words the condition that guarantees when there is a solution to $\boldsymbol{Ax} = \boldsymbol{b}$. Your answer should not be more than one sentence, and should mention one of the "4 fundamental subspaces" of $\boldsymbol{A}$.

   (c) Give an explicit condition involving b_1, b_2, and b_3 that guarantees when there is a solution to $\boldsymbol{Ax} = \boldsymbol{b}$.

(d) For each of the "the 4 fundamental subspaces" associated with $\boldsymbol{A}$ (the column space $C(\boldsymbol{A})$, row space $C(\boldsymbol{A}^T)$, nullspace $N(\boldsymbol{A})$, and left nullspace $N(\boldsymbol{A}^T)$), find a basis and compute its dimension. A basis for the subspace consisting of just the zero vector is the emptyset, $\varnothing$. [*Remark*: At this point, you may be able to determine some of these by inspection. If so, great! You do *not* need to show your work.]

(e) Find all solutions to $\boldsymbol{Ax} = \boldsymbol{b}$ when $\boldsymbol{b} = \begin{bmatrix} 0 \\ 2 \\ 8 \end{bmatrix}$.

5. Suppose $\boldsymbol{A}$ is a 2×3 matrix (2 rows and 3 columns) with rank $r = 2$.

   (a) Correctly complete the following sentence: The equation $\boldsymbol{Ax} = \boldsymbol{b}$ (always)(not always) has (a unique solution)(many solutions)(no solution).

   (b) Describe the four fundamental subspaes of $\boldsymbol{A}$ as best you can. Do *not* just state the definition of these spaces. [Descriptions like "all of $\boldsymbol{R}^n$", or "just the zero vector", or "*a line in* $\boldsymbol{R}^n$", or "*a plane in* $\boldsymbol{R}^m$" are fine, but use actual numbers, not just m or n.]

      i. The column space, $C(\boldsymbol{A})$.

      ii. The row space, $C(\boldsymbol{A}^T)$.

      iii. The nullspace, $N(\boldsymbol{A})$.

      iv. The left nullspace, $N(\boldsymbol{A}^T)$.