MthSc 311: Linear Algebra (Fall 2012)
Midterm 2
November 9, 2012

NAME:

Instructions

- Exam time is 50 minutes
- You may *not* use notes or books.
- Calculators are *not* allowed.
- **Show your work.** Partial credit will be given.

Question	Points Earned	Maximum Points
1		7
2		9
3		8
4		4
5		5
6		8
7		9
Total		**50**

Student to your left:

Student to your right:

1. (7 pts) Give an example of each of the following:

   (a) A set S of three nonzero vectors in $\mathbb{R}^3$ for which $S^\perp$ is a line.

   (b) An *orthonormal basis* for $\mathbb{R}^3$, including the vector $\boldsymbol{q}_1 = (1,1,1)/\sqrt{3}$.

   (c) A matrix that has no real-valued eigenvalues (that is, they are all complex numbers).

   (d) A 2×2 matrix with a repeated eigenvalue but only one (linearly independent) eigenvector.

   (e) A 2×2 matrix with a repeated eigenvalue but two (linearly independent) eigenvectors.

   (f) A non-orthogonal matrix that has orthonormal columns.

   (g) A non-zero matrix $\boldsymbol{A}$ and an eigenvector $\boldsymbol{v} \neq \boldsymbol{0}$ that is in its nullspace.

2. (9 pts) Fill in the blanks. No explanations needed, but think carefully before answering!

(a) If $\boldsymbol{Ax} = \boldsymbol{b}$ has no solution, then the closest vector to $\boldsymbol{b}$ in $C(\boldsymbol{A})$ is ____________________.

(either a formula or a description in words is fine).

(b) If $\boldsymbol{Ax} = \boldsymbol{b}$ has a solution $\boldsymbol{x}$, then the closest vector to $\boldsymbol{b}$ in $N(\boldsymbol{A}^T)$ is ____________________.

(c) Let $\widehat{\boldsymbol{x}}$ be the least-squares solution to $\boldsymbol{Ax} = \boldsymbol{b}$. Then $\boldsymbol{b} - \boldsymbol{A}\widehat{\boldsymbol{x}}$ is orthogonal to the

________________________ of $\boldsymbol{A}$.

(d) The formula for the projection matrix $\boldsymbol{P}$ onto the column space of a matrix $\boldsymbol{A}$ is $\boldsymbol{P} =$________________,

assuming that $\boldsymbol{A}$ has ________________ columns. In this case, $\boldsymbol{I} - \boldsymbol{P}$ is the projection

matrix onto the ________________________ of $\boldsymbol{A}$.

(e) If $\boldsymbol{Q}$ is an orthogonal matrix, then ________________ $= \boldsymbol{I}$ and the formula for the projection matrix onto the column space of $\boldsymbol{Q}$ in *simplified* form is $\boldsymbol{P} =$________________.

(f) If $\boldsymbol{v}$ is an eigenvector of $\boldsymbol{A}$ with eigenvalue λ, then $\boldsymbol{v}$ is in the nullspace of ________________.

3. (8 pts) Find the determinant of the following matrices:

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}, \qquad \boldsymbol{B} = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 2 & 3 & 0 \end{bmatrix}.$$

[*Hint*: Cofactor expansion will work to find $\det \boldsymbol{A}$, but it is certainly not necessary.]

4. (4 pts) Suppose a 4×4 matrix $\boldsymbol{A}$ has $\det \boldsymbol{A} = -1$. Find:

   (a) $\det(\frac{1}{2}\boldsymbol{A})$

   (b) $\det(-\boldsymbol{A})$

   (c) $\det(\boldsymbol{A}^2)$

   (d) $\det(\boldsymbol{A}^{-1})$

5. (5 pts) Use the Gram-Schmidt process to turn $\boldsymbol{a} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$ into an orthonormal set.

6. (8 pts) Find the eigenvalues and the eigenvectors of $\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$.

7. (9 pts) Consider the plane $x = y$ in $\mathbb{R}^3$. Note that this plane is the span of the set $S = \{\boldsymbol{v}_1, \boldsymbol{v}_2\}$ that contains the vectors $\boldsymbol{v}_1 = (1, 1, 0)$ and $\boldsymbol{v}_2 = (0, 0, 1)$.

   (a) Find a basis for $S^\perp$, the *orthogonal complement* of the plane $x = y$.

   (b) Project the vector $\boldsymbol{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ onto the plane $x = y$.

   (c) Project the vector $\boldsymbol{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ onto $S^\perp$. [*Hint*: Look at Problem 2.]