Read the following, which can all be found either in the textbook or on the course website.

- Chapters 1 & 2 of *Visual Group Theory* (VGT).
- VGT Exercises 1.1–1.4, 1.8–1.12, 2.13–2.17.
- The article *Group Think* by Steven Strogatz, which appeared in the NY Times in 2010.
- Francis Su’s essay on good mathematical writing.

Write up solutions to the following exercises.

1. Answer the following questions after reading Francis Su’s essay on good mathematical writing.
   
   (a) What is a good rule of thumb for what you should assume of your audience as you write your homework sets?
   (b) Is chalkboard writing formal or informal writing?
   (c) Why is the proof by contradiction on page 3 not really a proof by contradiction?
   (d) Name three things a lazy writer would do that a good writer would not.
   (e) What’s the difference in meaning between these three phrases?

   “Let \( A = 12. \)”
   “So \( A = 12. \)”
   “\( A = 12. \)”

2. Given a regular \( n \)-gon, let \( r \) be a rotation of it by \( 2\pi/n \) radians. This time, assume that we are not allowed to flip over the \( n \)-gon. These \( n \) actions form a group denoted \( C_n = \langle r \rangle = \{ e, r, r^2, \ldots, r^{n-1} \} \).

   (a) Draw a Cayley diagram for \( C_n \) for \( n = 4 \), \( n = 5 \), and \( n = 6 \).
   (b) For \( n = 4, 5, 6 \), find all minimal generating sets of \( C_n \).
   (c) Make a conjecture of what integers \( k \) does \( C_n = \langle r^k \rangle \) for a general fixed integer \( n \).

3. As we saw in lecture, the six symmetries of an equilateral triangle \( \triangle \) form a group denoted \( D_3 = \{ e, r, r^2, f, rf, r^2f \} \), where \( r \) is a 120° clockwise rotation and \( f \) is a flip about a vertical axis (which fixes the top corner). Since \( r \) and \( f \) suffice to generate all six of these symmetries, we write \( D_3 = \langle r, f \rangle \).

   (a) Let \( g \) be the reflection of the triangle that fixes the lower-left corner. Which of the six actions in \( D_3 \) is \( g \) equal to? Which action is \( fg \)?
   (b) Write all 6 actions of \( D_3 \) using only \( f \) and \( g \). Draw a Cayley diagram using \( f \) and \( g \) as generators.
   (c) To generate \( D_3 \), we need at least 2 actions. It is not difficult to show that if we have 3 generators, then one of them is unnecessary. Find all minimal generating sets of \( D_3 = \{ e, r, r^2, f, rf, r^2f \} \); note that all of them should have exactly two actions. Do not use \( g \) in this list.
4. The eight symmetries of a square \( \begin{array}{cc}
1 & 2 \\
4 & 3 \\
\end{array} \) form a group denoted \( D_4 \). Let \( r \) be a 90° clockwise rotation and \( f \) a horizontal flip (that is, about a vertical axis). It is not difficult to show that \( D_4 = \langle r, f \rangle \).

(a) Write all 8 actions of \( D_4 \) using \( r \) and \( f \) and draw a Cayley diagram using these two actions as generators.

(b) Let \( g \) be the reflection of the square that fixes the lower-left and upper-right corner. Which of the six actions in \( D_3 \) is \( g \) equal to? Which action is \( fg \)?

(c) Draw a Cayley diagram of \( D_4 \) using \( f \) and \( g \) as generators.

(d) Find all minimal generating sets of \( D_4 \). [Hint: There are 12.]

5. Pick any integer and consider this set of actions: adding any integer to the one you choose. This is an infinite set of actions; we might name them like “add 1” and “add \(-4210\),” etc. This is a group. Find all minimal generating sets. Sketch a Cayley graph for this group using one of these minimal generating sets.