1. Consider the reactions where two substrates $S$ and $T$ compete for binding to an enzyme $E$ to produce two different products $P$ and $Q$:

$$E + S \overset{p_1}{\underset{p_2}{\rightleftharpoons}} ES \overset{p_3}{\rightarrow} P + E$$
$$E + T \overset{q_1}{\underset{q_2}{\rightleftharpoons}} ET \overset{q_3}{\rightarrow} Q + E$$

(a) Assuming that each reaction follows the Michaelis-Menten kinetics, derive rate equations for $P$ and $Q$ in this system. That is, determine $d[P]/dt$ and $d[Q]/dt$.

(b) Explain the effects of the competition occurring.

2. The Hill equation is an approximation for multi-molecule binding and it assumes simultaneous binding of $n$-molecules of a substrate $S$ to the enzyme $E$. Suppose that two molecules of the substrate $S$ are undergoing a reaction with an enzyme in an ordered manner as follows:

$$E + S \overset{k_1}{\underset{k_2}{\rightleftharpoons}} ES + S \overset{k_3}{\underset{k_4}{\rightleftharpoons}} ES_2 \overset{k_5}{\rightarrow} P + E.$$  

(a) Derive a rate equation under the steady state assumption and compare it with the Hill equation with Hill coefficient $n = 2$:

$$\frac{d[P]}{dt} = \frac{V_{\text{max}}[S]^2}{K_m + [S]^2}.$$  

(b) When do these two equations become roughly the same?