1 Modeling (overview)

Models are everywhere!

Physics	Psychology
Chemistry	Sociology
Engineering	Statistics
Economics	Meteorology
Biology	

What is a model?

Ex: Physics Ball drops from a roof.

$x'' = -9.8, \quad x' = -9.8t, \quad x = -4.9t^2$

Finance Rate of an investment grows at a rate proportional to its quantity (e.g., $r = 5\%$.)

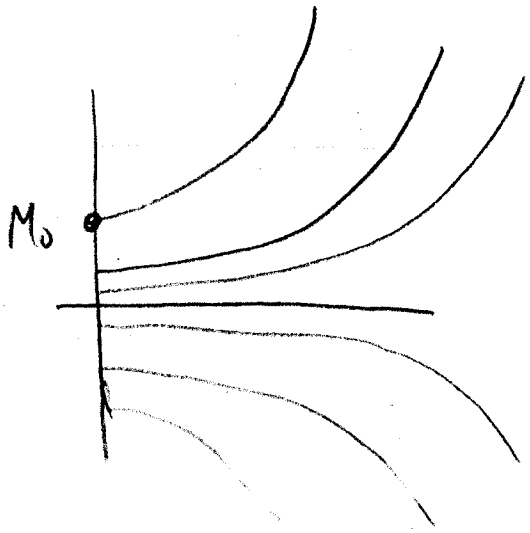
Eqn: $P' = .05P$ $\overset{\text{MthSc 208}}{\rightsquigarrow}$ $P(t) = P_0 e^{.05t}$

Biology A colony of bacteria grows at a rate proportional to its size.

Eqn: $M' = kM$ $\overset{208}{\rightsquigarrow}$ $M(t) = M_0 e^{kt}$

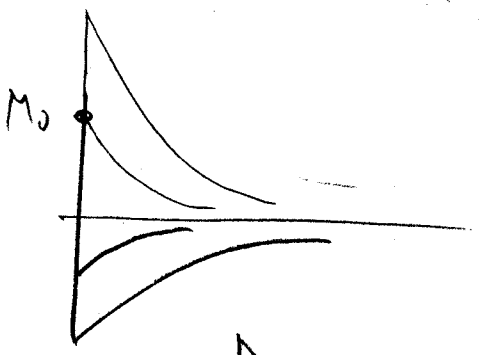
(2)

Chemistry A radioactive substance decays at a rate proportional to how much is remaining.



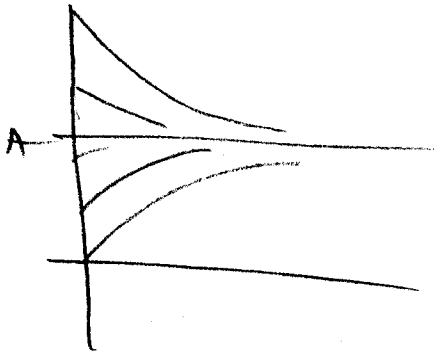
Eg: $M' = -kM$

$M(t) = M_0 e^{-kt}$



Physics The temp of a cup of coffee cools at a rate proportional to

(temp of coffee) − (ambient temp, A)



Eg: $T' = k(A - T)$ $\quad T(t) = A + Ce^{-kt}$

Goals with these models

- Analyze them (initial value? limiting value)
- Validate them (does the math reflect the actual phenomenon?)

* More complicated models:

Falling object w/o air resistance: $F = ma = -mg$

" " w/ " " : $F = -mg + R(v)$

What is $R(v)$? Try $R(v) = -rv$ (good approx.)

$$F = ma = mv' = -mg - rv \implies v' = -g - \frac{r}{m}v$$

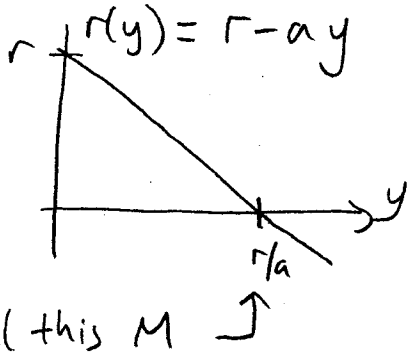
Soln: $v(t) = -\frac{mg}{r} + Ce^{-\frac{r}{m}t}$

Note: Init. vel: $v(0) = -\frac{mg}{r} + C$

Termin. vel: $\lim_{t\to\infty} v(t) = -\frac{mg}{r}$.

Population growth

Logistic eqn: $y'(t) = r(y)\cdot y(t)$ ← was just r for exp. growth.



call this M

$r(y)$ should be decreasing.

$$y' = y(r - ay) = ry\left(1 - \frac{y}{M}\right)$$

Soln: $y(t) = \frac{M}{1 + Ce^{-rt}}$

Init pop: $y(0) = \frac{M}{1+C}$

Limiting pop: $\lim_{t\to\infty} y(t) = M$.

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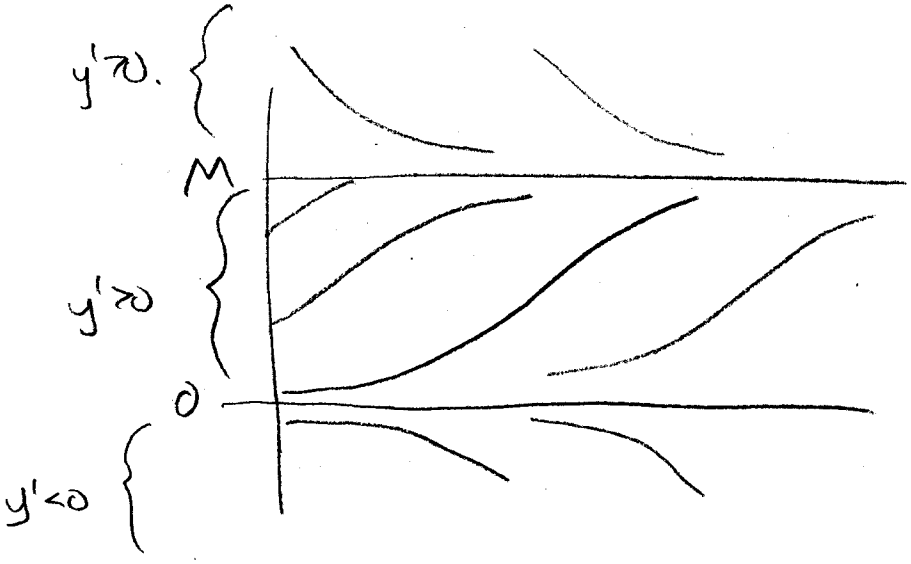
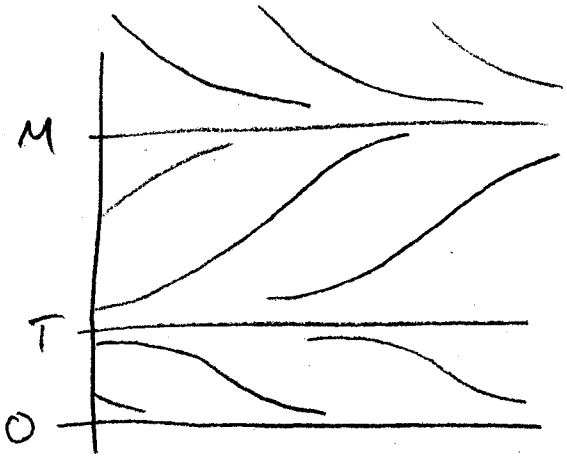


Two "steady-states":

$y(t) = 0, M.$

Threshold eg'n:



Want steady-states

$y(t) = 0, M, T:$

$$y' = -ry\left(1 - \frac{y}{M}\right)\left(1 - \frac{y}{T}\right)$$

This modeled the passenger pigeon quite well!

Note: There is a lot of ecology / biology behind the scenes:

- Gestation period
- # of offspring
- Geography
- reproduction (sexual vs. asexual)
- threshold?
- predator?