Read: Lax, Chapter 4, pages 32–43.

1. Let $T: X \rightarrow U$, with $\dim X = n$ and $\dim U = m$. Show that there exist bases $B$ for $X$ and $B'$ for $U$ such that the matrix of $T$ in block form is

$$M = \begin{bmatrix} I_k & 0 \\ 0 & 0 \end{bmatrix}$$

where $I_k$ is the $k \times k$ identity matrix, and the other blocks are either empty or contain all zeros.

2. Consider the linear map $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ with matrix representation

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & -2 \\ -3 & 0 & 3 \end{bmatrix}$$

with respect to the standard basis. What is the matrix representation of $T$ with respect to the basis $\{(1, -1, 0), (0, 1, -1), (1, 0, 1)\}$?

3. Let $P_n$ be the vector space of all polynomials over $\mathbb{R}$ of degree less than $n$.

   (a) Show that the map $T: P_3 \rightarrow P_4$ given by

   $$T(p(x)) = 6 \int_1^x p(t) \, dt$$

   is linear. Indicate whether it is $1$–$1$ or onto.

   (b) Let $B_3 = \{1, x, x^2\}$ be a basis for $P_3$ and let $B_4 = \{1, x, x^2, x^3\}$ be a basis for $P_4$. Find the matrix representation of $T$ with respect to these bases.