Math 4500 Worksheet: RNA folding
April 2015

A *langauge* consists of a set of finite strings that can be constructed from an alphabet Σ of *terminal symbols* (lowercase) and "temporary" *nonterminal symbols* (uppercase), according to *production rules.*

In a *context free grammar* (CFG), all rules have the form

$$A \longrightarrow \alpha A \beta\,,$$

where α and β are strings (possibly empty).

A *derivation* of a string is a set of steps that creates it from the start symbol S. A *left derivation* is one where rules are always applied to nonterminals in a left-to-right order. A right derivation is defined similarly.

Every derivation can be visualized using a *parse tree.*

Exercises.

(1) Construct a regular grammar that generates the language $\{b^n a \mid n \geq 0\}$. Try to construct a regular grammar that generates the language $\{ab^n a \mid n \geq 0\}$. What goes wrong?

(2) Consider the following grammar:

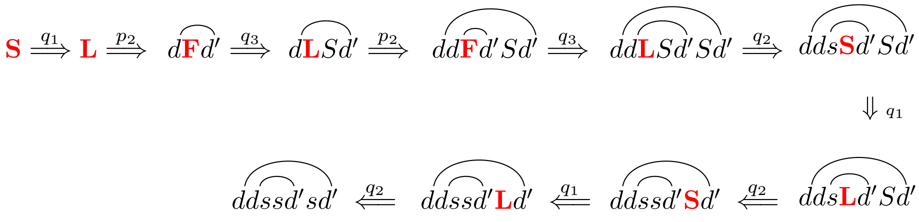
$$S \longrightarrow SS|a\,.$$

Show that this grammar is *ambigious* by finding two left derivations of the string $\alpha = aaa$ that have different parse trees.

(3) The Knudsen-Hein grammar is a *stochastic context free grammar* (SCFG) defined by the following production rules:

$$\begin{aligned} S &\longrightarrow LS\ (p_1)\,|\,L\ (q_1) \\ L &\longrightarrow dFd'\ (p_2)\,|\,s\ (q_2) \\ F &\longrightarrow dFd'\ (p_3)\,|\,LS\ (q_3) \end{aligned}$$

Below is a *left derivation* of the string $\alpha = ddssd'sd'$:



S $\xRightarrow{q_1}$ L $\xRightarrow{p_2}$ dFd' $\xRightarrow{q_3}$ $dLSd'$ $\xRightarrow{p_2}$ $ddFd'Sd'$ $\xRightarrow{q_3}$ $ddLSd'Sd'$ $\xRightarrow{q_2}$ $ddsSd'Sd'$
$\Downarrow q_1$
$ddssd'sd'$ $\xLeftarrow{q_2}$ $ddssd'Ld'$ $\xLeftarrow{q_1}$ $ddssd'Sd'$ $\xLeftarrow{q_2}$ $ddsLd'Sd'$

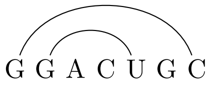
(a) Construct a *parse tree* for $\alpha = ddssd'sd'$.

(b) Compute the right derivation of the same string, $\alpha = ddssd'sd'$ and draw the corresponding (right) parse tree.

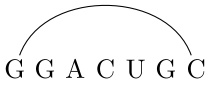
(4) Use the Knudsen-Hein grammar to construct a derivation the hairpin loop $ssddsssd'd'ss$, and compute its probability.

(5) Modify the rules to make the minimum loop size $j - i \geq 4$ and repeat the above problem.

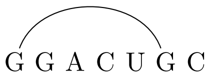
(6) Allowing arc lengths of length $\lambda = 3$, there 7 legal folds of the sequence $\mathbf{b}$ = GGACUGC. One of these is the trivial unfolded structure. The other 6 are shown below:



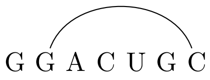
$P(S_1) = p_2^2 q_1^3 q_2^3 q_3^2$



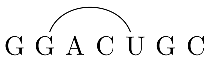
$P(S_2) = p_1^3 p_2 q_1^2 q_2^5 q_3$



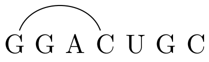
$P(S_3) = p_1^3 p_2 q_1^2 q_2^5 q_3$



$P(S_4) = p_1^3 p_2 q_1^2 q_2^5 q_3$



$P(S_5) = p_1^3 p_2 q_1^2 q_2^5 q_3$



$P(S_6) = p_1^3 p_2 q_1^2 q_2^5 q_3$

Find a derivation for each of these using the Knudsen Hein grammar and construct its parse tree.

(7) Consider the following "mystery grammar" from (Durbin, 1998):

$$\begin{aligned} S &\longrightarrow aAu \,|\, cAg \,|\, gAc \,|\, uAa \\ A &\longrightarrow aBu \,|\, cBg \,|\, gBc \,|\, uBa \\ B &\longrightarrow aCu \,|\, cCg \,|\, gCc \,|\, uCa \\ C &\longrightarrow gaaa \,|\, gcaa. \end{aligned}$$

What is the language L derived from this grammar? Describe it in terms of RNA secondary structures.