1. Consider a structured population model with matrix $P = \begin{bmatrix} .3 & 2 \\ .4 & 0 \end{bmatrix}$ (called a *Leslie matrix*):

   (a) By thinking about the biological meaning of each entry in this matrix, do you think it describes a growing or declining population. Would you guess the population size would change rapidly or slowly? Explain your reasoning.

   (b) Compute the eigenvalues and eigenvalues of the model. (Use a computer.)

   (c) Express the initial vector $\boldsymbol{x}_0 = (5, 5)$ as a sum of the eigenvectors.

   (d) Use your answer in the previous part to give a formula for the population vector $\boldsymbol{x}_t$.

   (e) What is the long-term behavior, $\lim_{t \to \infty} \boldsymbol{x}_t$?

2. A model given in (Cullen, 1985), based on data collected in (Nellis and Keith, 1976), describes a certain coyote population. The population is stratified in three classes: pup, yearling, and adult, and the matrix

$$P = \begin{bmatrix} .11 & .15 & .15 \\ .3 & 0 & 0 \\ 0 & .6 & .6 \end{bmatrix}$$

   describes changes over a time step of 1 year.

   (a) Carefully explain what each entry in this matrix is saying about the population.

   (b) Find the intrinsic growth rate (dominant eigenvalue) and corresponding eigenvector. Feel free to use a computer.

   (c) Will the population grow or decline? Quickly or slowly?

3. In class, we saw that the model $\begin{cases} P_{t+1} = P_t(1 + 1.3(1 - P_t)) - .5P_tQ_t \\ Q_{t+1} = .3Q_t + 1.6P_tQ_t \end{cases}$

   has a steady-state equilibrium that is approached through oscillations. Because the discrete logistic model $P_{t+1} = P_t(1 + 1.3(1 - P_t))$ on which it is based has $r = 1.3$, we know that it alone would produced underdamped dynamics (=damped oscillations) rather than the overdamped dynamics that arise when $r < 1$. Thus, it is not clear whether the oscillations in the model above are inherient to the model or, simply due to $r > 1$.

   Explore this using the MATLAB program `twopop` with a number of values of r – less than and greater than 1.3 in the predator–prey model. Can find a value or $r < 1$ that yields oscillations in the predator–prey model? If so, can you find a value of r that yields no oscillations, and where is the "threshold" between these two dynamical regimes? Are there any other "thresholds" where the qualitative dynamics changes? Include a print-outs or detailed sketches for a few different values of r.

4. Imagine a predator–prey interaction in which a certain number of the prey population cannot be eaten because of a refuge in their environment that the predator cannot enter.

   (a) Give an real-life example two populations that might exhibit this feature.

(b) Why might interaction terms like $-s(P-w)Q$ and $v(P-w)Q$ be reasonable in the modeling equation?

(c) What is the meaning of w? Would you expect $w > P$ or $w < P$ to be more reasonable?