# Math 8530: Linear Algebra

Spring 2017

Martin M-203, TR 12:30–1:45

**Instructor** Dr. Matt Macauley (macaule@clemson.edu)
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**Textbook** *Linear Algebra and its Applications*, by Peter Lax.

**Prerequisites** Math 3110 (Linear Algebra) or 4530 (Real Analysis).

**Policies**

- Homework assignments will accumulate from lecture to lecture and will be due roughly once a week. I will post the problems on my website. Late homework will *not* be accepted.
- Attendance on test days is mandatory. Class attendance at other times is not mandatory, but is strongly recommended.
- If you get an A on the final exam, then you get an A in the course.
- All drop/add procedures are your responsibility.
- Absent Professor Policy: If the instructor has not arrived within 10 minutes of the scheduled class time, you may assume that class has been canceled.
- All use of cell phones, laptops, and PDAs is prohibited during lecture. Calculators, cell phones, laptops, and PDAs will not allowed during exams.
- Cell phone policy: http://www.youtube.com/watch?v=FYwpxU_G4Z0
- I will NOT post homework solutions. However, I will gladly help you with any of the problems during office hours or whenever I'm around.
- No whining.

**Learning Outcomes** This course will be a comprehensive survey of linear algebra at the graduate level. The goals are two-fold – to prepare you for the linear algebra part of the prelim, and to build up a solid linear algebra foundation necessary for your other classes and your research, as Linear algebra is a beautiful subject that appears in nearly all areas of mathematics, *especially* in applied fields such as statistics, operations research, and computational mathematics.

**Grading** The final grade will be calculated as follows:

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| Homework: | 20% |
| Midterm 1: | 20% |
| Midterm 2: | 20% |
| Final exam: | 40% |

**Homework** Homework assignments will accumulate from lecture to lecture and will be due several times a week. I will post the assignments on my website, as I like to make all materials freely available to everybody (Warning: Websites such as *Course Hero* are a SCAM!). Students can collaborate on their homework problems, but they *must* write up and submit their homeworks separately. Late homeworks will **not** be accepted. You are encouraged to typeset your homework assignments (LaTeXpreferred but not required). You should keep all the graded homeworks in case of missing grades due to missing name or typo errors.

Topics

**Section 1**: Linear algebra fundamentals [chapter 1 of Lax]

- Vector spaces
- Linear maps
- Subspaces
- Spanning, independence, bases, and dimension
- Complementary subspaces and direct sums
- Direct products
- Finite vs. infinite products
- Quotient spaces
- Application: solving linear ODEs

**Section 2**: Duality [chapter 2 of Lax]

- Linear functionals & the dual space
- Scalar product notation
- Annihilators
- Double duals

**Section 3**: Linear mappings [chapter 3 of Lax]

- Range & nullspace
- Rank-nullity theorem
- Appliation: systems of linear equations
- Application: polynomial interpolation
- Application: average values of polynomials over intervals
- Application: numerial solutions to Laplace's equation (finite differences)
- Algebra of linear mappings
- Transposes (as mappings between dual spaces)

**Section 4**: Matrices [chapter 4 of Lax]

- How a choice of basis determines the matrix of a linear map
- 4 ways to multiply matrices (row-by-cols, by rows, by cols, col-by-rows)
- The matrix of a transpose map
- Column rank & row rank
- Change of basis & similar matrices
- Systems of equations and Gaussian elimination

**Section 5**: Determinant and trace [Halmos' book + Strang's book + chapter 5 of Lax]

- Geometric idea of determinant

- Permutations & discriminant
- Multilinear forms
- Symmetric, skew-symmetric, and alternating k-linear forms
- The vector space of alternating n-linear forms is 1-dimensional
- Basis-free definition & universal property of the determinant
- Determinants & matrices (Laplace expansion)
- Cramer's rule
- Trace

**Section 6**: Spectral theory [chapter 6 of Lax + appendix 15 of Lax + supplimental]

- Eigenvectors & eigenvalues
- Distinct eigenvalues lead to linearly independent eigenvectors
- Spectral mapping theorem: Eigenvalues of $A$ vs. $q(A)$.
- Cayley-Hamilton theorem
- Algebraic multiplicity vs. geometric multiplicity of eigenvalues
- Generalized eigenvectors
- Spectral theorem: there is always a full set of generalized eigenvectors
- Minimal polynomials and Jordan canonical form
- Commuting maps and simulataneous diagonaliziability
- Application: systems of ODEs & matrix exponentials

**Section 7**: Euclidean structure (inner product spaces) [chapter 7 of Lax]

- Review of Euclidean structure (length, dot product, orthogonality, angles)
- Real inner product spaces
- Cauchy-Schwarz inequality
- Triangle inequality & Pythagorean theorem
- Orthonormal bases
- Gram-Schmidt process & QR-factorization
- Identification of a space with its dual
- Orthogonal complement & projections
- Adjoints
- Application: least squares
- Isometries & orthogonal matrices
- Norms of linear maps
- The subset of invertible maps is open
- Basic analysis review (convergence, Cauchy sequences, completeness, local compactness)
- An inner product spaces is locally compact iff it is finite-dimensional
- Complex inner product spaces
- Complex inner product, orthogonality, and unitary maps

- Application: Fourier series (real and complex)

**Section 8**: Self-adjoint mappings [chapter 8 of Lax]

- Decomposition of a linear map into a self-adjoint plus an anti-self-adjoint map
- Motivation: 2nd order Taylor approximations & the Hessian
- Quadratic forms
- Self-adjoint maps have real eigenvalues and a full set of orthonormal eigenvectors
- Projections onto eigenspaces and spectral resolutions
- Self-adjoint commuting maps have a common spectral resolution
- Anti-self-adjoint maps have purely imaginary eigenvalues and a full set of orthonormal eigenvectors
- Normal maps
- Unitary maps
- The Rayleigh quotient & its critical points
- Minmax principle for the eigenvalues of a self-adjoint map
- Positive definite mappings and the generalized Reyleigh quotient
- Application to numerical linear algebra: 2nd order Taylor approximation of eigenvalues
- Properties of $A^*A$.

**Section 9**: Positive(-definite) mappings [chapter 10 of Lax]

- Tensor product of two vector spaces (4 different ways to think of it)
- Basic properties of positive and non-negative mappings
- A partial order on on the set of self-adjoint maps
- Symmetrized products
- Monotone matrix functions (MMFs)
- A functional analysis characterization of all MMFs
- Gram matrices & non-standard inner products
- Schur's theorem of positive matrices
- Singular value decomposition
- Right, left, and pseudo-inverses

**Extra topics** (since I'll probably finish a few days early) [misc. sources]

- Cyclic subspaces
- Companion matrices and rational canonical form
- What is a module? (A vector space over a ring)
- How Jordan & rational canonical form generalize from vector spaces to modules
- Avoidance of crossings (the space of singular self-adjoint mappings has codimension 2).

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| Key Dates | January 11 (Wed) | First day of class. |
| | January 18 (Wed) | Last day to register or add a class |
| | January 16 (Mon) | Holiday: MLK Day |
| | January 25 (Wed) | Last day to drop a class or withdraw from the University without a W grade |
| | March 17 (Fri) | Last day to drop a class or withdraw from the University without final grades |
| | March 20–24 (M–F) | Spring Break |
| | April 27 (Thu) | Last day of class |
| | May 1 (Mon) | Final exam 3:00–5:30pm |

**The official statement on Academic Integrity**

As members of the Clemson University community, we have inherited Thomas Green Clemson's vision of this institution as a *high seminary of learning.* Fundamental to this vision is a mutual commitment to truthfulness, honor, and responsibility, without which we cannot earn the trust and respect of others. Furthermore, we recognize that academic dishonesty detracts from the value of a Clemson degree. Therefore, we shall not tolerate lying, cheating, or stealing in any form.

When in the opinion of a faculty member, there is evidence that a student has committed an act of academic dishonesty, the faculty member shall make a formal written charge of academic dishonesty including a description of the misconduct, to the Dean of the Graduate School. At the same time, the faculty member may, but is not required to, inform each involved student privately of the nature of the alleged charge.