Lecture 1.3: Approximating Solutions to Differential Equations

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Motivation (from single variable calculus)

Classic calculus problem

Suppose $f(1) = 1$ and $f'(1) = 1/2$. Use the tangent line to $f(x)$ at $x = 1$ to approximate $f(1.5)$.

Old vs. New

Classic calculus problem

Suppose $f(1) = 1$ and $f'(1) = 1/2$. Use the tangent line to $f(x)$ at $x = 1$ to approximate $f(1.5)$.

New differential equation problem

Consider the ODE $y' = y - t$, and say $y(1) = 1$. Can we approximate $y(1.5)$?

Euler's method

Example

Suppose $y(t)$ solves the ODE $y' = y - t$, and $y(1) = 1$. Use Euler's method to approximate $y(1.5)$.

Euler's method

Summary

Given $y' = f(t, y)$ and $y(t_0) = y_0$ with a stepsize h:

$$\begin{aligned}
(t_1, y_1) &= \big(t_0 + h, y_0 + f(t_0, y_0) \cdot h\big) \\
(t_2, y_2) &= \big(t_1 + h, y_1 + f(t_1, y_1) \cdot h\big) \\
&\vdots \\
(t_{k+1}, y_{k+1}) &= \big(t_k + h, y_k + f(t_k, y_k) \cdot h\big)
\end{aligned}$$