Lecture 2.2: Initial value problems

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Math 2080, Differential Equations

Introduction

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Example 1

Solve the IVP $y' = k(72 - y)$; $y(0) = 90$.

Word problems

Example 2 (exponential growth)

A house sells in 2003 for $179,500 and was on sale in 2008 for $319,500.

(a) What was the average rate of appreciation of the value?

(b) Suppose the market has been increasing at a 9% rate. How much is the house worth?

Word problems

Example 3 (exponential decay)

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(a) What is the half-life?

(b) How long until only 1 gram remains?

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Example 4 (exponential decay to a value)

My coffee is $120°$ when class starts, and the classroom is $75°$. After 30 minutes, the coffee is $100°$.

(a) What will the temperature be at the end of class ($t = 50$)?

(b) Suppose it was brewed at $160°$. When did I brew it?

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