Lecture 2.5: Linear differential equations

Matthew Macauley

Department of Mathematical Sciences
Clemson University
http://www.math.clemson.edu/~macaule/

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Motivation

Recall

A first order ODE is linear if it can be written as

$$y'(t) + a(t)y(t) = f(t),$$

and moreover, is homogeneous if $f(t) = 0$.

Linear differential equations and their solutions have a lot of structure.

Understanding the structure helps demystify these objects and reveals their simplicity.

We will see two "Big Ideas" in this lecture, and these will re-appear when we study 2nd order ODEs.

Along the way, we will uncover a neat short-cut for solving ODEs that is usually not covered in a differential equation course.

Big idea #1: homogeneous ODEs

Big idea 1

Suppose a homogeneous ODE $y' + a(t)y(t) = 0$ has solutions $y_1(t)$ and $y_2(t)$. Then

$$C_1 y_1(t) + C_2 y_2(t)$$

is a solution for any constants C_1 and C_2.

Big idea #2: inhomogeneous ODEs

Big idea 2

Consider an inhomogeneous ODE $y' + a(t)y(t) = f(t)$. If $y_p(t)$ is *any* particular solution, and $y_h(t)$ is the general solution to the related "homogeneous equation", $y' + a(t)y = 0$, then the general solution to the inhomogeneous equation is

$$y(t) = y_h(t) + y_p(t)\,.$$

A nice shortcut

Applications of $y = y_h + y_p$

- Solving for $y_h(t)$ is usually easy (separate variables).
- Sometimes, it's easy to find some $y_p(t)$ by inspection.
- When this happens, we automatically have the general solution!

Example 1

Solve $T' = k(72 - T)$.

Exploiting our shortcut

Example 2

Solve $y' = 2y + t$.

Exploiting our shortcut

Example 3

Solve $y' = 2y + e^{3t}$.

An interesting observation