Lecture 2.7: Advanced mixing problems

Matthew Macauley

Department of Mathematical Sciences
Clemson University
http://www.math.clemson.edu/~macaule/

Math 2080, Differential Equations

Motivation

Last time

Suppose we have a tank of fresh water.

- Salt water flows IN at some (constant) rate.
- The water in the tank is fully mixed.
- Water drains OUT of the tank at the same rate.

Question: What is the concentration of salt in the tank at time t?

This time

What if. . .

- The incoming and outgoing rates are different?
- There are two tanks, and one drains into the other.

Rate in $\neq$ Rate out

Example 2

Suppose we have a tank containing 150 gallons of fresh water.

- Salt water (concentration: 2 oz/gal) flows in at 3 gal/min.
- The water in the tank is fully mixed.
- Water drains from the tank at 1 gal/min.

Question: What is the concentration of salt in the tank the moment it overflows?

First step (always!)

Let $x(t) = \#$ ounces of salt in the tank at time t. Then

$$x'(t) = \text{ (rate in) } - \text{ (rate out)}.$$

Example 2 (cont.)

Two tanks

Example 3

Suppose we have two tanks, A and B.

- Tank A contains 100 gallons and 20 oz of salt.
- Tank B contains 200 gallons and 40 oz of salt.
- Fresh water enters tank A at 5 gal/min.
- Tank A drains into tank B at 5 gal/min.
- Tank B drains at 5 gal/min.

Question: How much salt is there in each tank at time t?

Example 3 (cont.)