Lecture 3.2: Equations with constant coefficients

Matthew Macauley

Department of Mathematical Sciences
Clemson University
http://www.math.clemson.edu/~macaule/

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Introduction

Recall

A linear 2nd order ODE has the form $y'' + p(t)y' + q(t)y = f(t)$, and it is homogeneous if $f(t) = 0$.

Approach

We will *always* solve the related "homogeneous equation" first. In this lecture, we will consider homogeneous ODEs for which $p(t)$ and $q(t)$ are constants. The general solution will be

$$y(t) = C_1 y_1(t) + C_2 y_2(t).$$

Goal: Find any $y_1(t)$ and $y_2(t)$ that solve the ODE.

Example 1

Find the general solution to $y'' = k^2 y$.

Example 2

Find the general solution to $y'' = -k^2 y$.

More examples

Example 3

Find the general solution to $y'' - 3y' + 2y = 0$.

A problem case

Example 4

Find the general solution to $y'' - 6y' + 9y = 0$.

Another problem case

Example 5

Suppose we want to solve $y'' + py' + qy = 0$, and the roots of the characteristic equation are complex numbers $r_{1,2} = a \pm bi$, with $b \neq 0$.

A review of complex numbers and Euler's formula