# Lecture 3.4: Simple harmonic motion

Matthew Macauley

Department of Mathematical Sciences
Clemson University
http://www.math.clemson.edu/~macaule/

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# Introduction

## Mass-spring systems

If $x(t)$ is the displacement of a mass $m$ on a spring, then $x(t)$ satsifies

$$mx'' + 2cx' + \omega_0^2 x = f(t)\,,$$

where

- $c$ is the damping constant
- $\omega_0$ is frequency
- $f(t)$ is the external driving force

# Harmonic motion: $mx'' + 2cx' + \omega_0^2 x = f(t)$

## Simple harmonic motion

When there is no damping or driving force, then the mass exhibits simple harmonic motion:

$$x'' + kx = 0\,, \qquad k = \omega^2 > 0\,.$$

# A better way to write the solution to $x'' + \omega^2 x = 0$

## Big idea

Any function $x(t) = a\cos(\omega t) + b\sin(\omega t)$ can be written *as a single cosine wave*

$$x(t) = A\cos(\omega t - \phi) = A\cos\left(\omega\big(t - \frac{\phi}{\omega}\big)\right)$$

with

- Amplitude $A = \sqrt{a^2 + b^2}$
- Phase shift $\phi/\omega$, where "$\phi = \tan^{-1}(b/a)$."

# Simple harmonic motion with an external force

## Example

A 2 kg mass is suspended from a spring. The displacement of the spring once the mass is attached is 0.5 meters. If the mass is displaced 0.12m downward from equilibrium, set up and solve the initial value problem that models this.