Lecture 5.1: What is a Laplace transform?

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Overview

Applications

Laplace transforms are:

- Used to solve and analyze linear ODEs.
- Useful when the forcing term is discontinuous.

Definition and example

Definition

Suppose $f(t)$:

- is defined for $0 < t < \infty$;
- doesn't grow too fast: $|f(t)| \leq Ce^{at}$ for some C and a.

Then the Laplace transform of f is the function $\mathcal{L}(f)$, where

$$\mathcal{L}\{f(t)\}(s) = \int_0^\infty f(t)e^{-st}\,dt\,.$$

Example 1

Find the Laplace transform of $f(t) = e^{at}$.

More examples

Example 2

Find the Laplace transform of $f(t) = t$.

More examples

Other common functions

- $\mathcal{L}\{t^n\} = \dfrac{n!}{s^{n+1}}$;
- $\mathcal{L}\{\sin bt\} = \dfrac{b}{s^2 + b^2}$;
- $\mathcal{L}\{\cos bt\} = \dfrac{s}{s^2 + b^2}$;

Piecewise functions

Example 3

Compute the transform of the piecewise function $f(t) = \begin{cases} 1, & 0 \leq t < 1 \\ 0, & t \geq 1 \end{cases}$

More examples

Example 4

Compute the transform of the piecewise function $f(t) = \begin{cases} t, & 0 \leq t < 1 \\ 1, & t \geq 1 \end{cases}$