Lecture 5.5: Impulse functions

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Math 2080, Differential Equations

## Motivation

### Question

What is the derivative of the Heavyside function?

# The Laplace transforms of a periodic function

## Example

The Dirac delta function centered at $p$ is $\delta_p(t) = \lim\limits_{\epsilon \to 0} \delta_p^{\epsilon}(t) = \begin{cases} 0, & t \neq p \\ \infty, & t = p \end{cases}$

## Properties

(i) $\displaystyle\int_{-\infty}^{\infty} \delta_p(t)\, dt = 1$.

(ii) $\mathcal{L}\{\delta_0(t)\} = 1$.

# Differential equations with impulse forcing terms

### Example

Solve the IVP: $y'' + 2y' + 2y = \delta_0(t)$, $y(0) = 0$, $y'(0) = 0$.