Lecture 6.2: Computing Fourier series

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Last time

Formula for the Fourier coefficients

Let $f(x)$ be a piecewise continuous 2π-periodic function. We can write

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx,$$

where

$$a_n = \langle f(x), \cos nx \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$b_n = \langle f(x), \sin nx \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx.$$

Computations

Example 1: square wave

Find the Fourier series of the 2π-periodic function $f(x) = \begin{cases} 1, & 0 \leq x < \pi \\ -1, & \pi \leq x < 2\pi \end{cases}$

Computations

Example 2: sawtooth wave

Find the Fourier series of the 2π-periodic function defined by $f(x) = x$ on $(-\pi, \pi]$.