Lecture 6.5: Applications of Fourier series

Matthew Macauley

Department of Mathematical Sciences
Clemson University
http://www.math.clemson.edu/~macaule/

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Introduction

Parseval's identity

If $f(x) = \dfrac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$, then

$$\langle f(x), f(x) \rangle := \frac{1}{\pi} \int_{-\pi}^{\pi} \big(f(x)\big)^2 dx = \frac{1}{2} a_0^2 + \sum_{n=1}^{\infty} a_n^2 + b_n^2 \,.$$

An application of Parseval's identity

Sum of inverse squares

Compute the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots$.

Solving ODEs with Fourier series

Example

Solve the ODE $x'' + \omega^2 x = f(t)$ where $f(t)$ is the 2π-periodic sawtooth wave defined by $f(t) = t$ on $(-\pi, \pi]$.