Math 2080: Differential Equations
Worksheet 3.4: Simple harmonic motion

NAME:

When a 3 kg mass is hung from a spring, the spring is displaced by 1 m. Now, suppose that the mass is hanging from rest and then given a sharp jolt downwards so its initial velocity is 1 m/sec. We will set up and solve an initial value problem that models this.

(a) Sketch this mass-spring system, before and after the mass is place on the spring. (Let $x = 0$ be the height of the spring without the mass).

(b) At equilibrium, the spring force kx_0 equals the gravitational force, mg, in magnitude. Use this to solve for the spring constant k.

(c) Newton's 2$^{\text{nd}}$ law tells us that $F = mx''$, which is equal to the sum of the forces (gravitational and spring). Write down a second-order differential equation that models this. Include both initial conditions, $x(0)$ and $x'(0)$.

(d) Find the general solution to this ODE. Is there a steady-state solution? If so, describe it.

(e) Solve the initial value problem.