Math 2080: Differential Equations
Worksheet 3.5: Damped harmonic motion

NAME:

1. For the following exercises, rewrite the given function in the form

$$y = A\cos(\omega t - \phi) = A\cos\left(\omega\big(t - \frac{\phi}{\omega}\big)\right),$$

and then plot the graph of this function.

(a) $y = 3\cos 2t + 4\sin 2t$

(b) $y = 3\cos 2t - 4\sin 2t$

(c) $y = -3\cos 2t - 4\sin 2t$

2. Consider the undamped oscillator

$$mx'' + kx = 0, \qquad x(0) = x_0, \qquad x'(0) = v_0.$$

Write the general solution of this initial value problem in the form $x(t) = a\cos\omega t + b\sin\omega t$ (i.e., determine a, b, and ω.), and then write it in the form $x(t) = A\cos(\omega t - \phi)$ (i.e., determine A).

3. The function $x(t) = \cos 6t - \cos 7t$ has mean frequency $\bar{\omega} = 13/2$ and half difference $\delta = 1/2$. Thus,

$$\cos 6t - \cos 7t = \cos\left(\tfrac{13}{2} - \tfrac{1}{2}\right)t - \cos\left(\tfrac{13}{2} + \tfrac{1}{2}\right)t = 2\sin\tfrac{1}{2}t\ \sin\tfrac{13}{2}t\,.$$

Use a computer or calculator to plot both $f(t) = \cos 6t - \cos 7t$ and the "envelope" $g(t) = 2\sin\frac{1}{2}t$ on the same set of axes.

4. Let $\omega_0 = 11$. Use a computer to plot the graph of the function

$$x(t) = \frac{\cos\omega t - \cos\omega_0 t}{\omega_0^2 - \omega^2}$$

for $\omega = 9$, 10, 10.5, 10.9, and 10.99 on the time interval $[0, 24]$. Explain how these solutions approach the resonance solution as $\omega \to \omega_0$.