# Math 2080: Differential Equations Worksheet 3.6: Variation of parameters 

NAME:

Consider the inhomogeneous differential equation $y^{\prime \prime}+9 y=\sec 3 t$.
(a) Solve the related homogeneous equation, $y_{h}^{\prime \prime}+9 y_{h}=0$, by inspection.
(b) If $y_{h}(t)=C_{1} y_{1}(t)+C_{2} y_{2}(t)$ is your answer to the previous part, then assume that there is a particular solution to the original ODE of the form $y_{p}=v_{1} y_{1}+v_{2} y_{2}$. Compute $y_{p}^{\prime}$.
(c) The $y_{p}^{\prime}$ from the previous part should have four terms; one each with $v_{1}, v_{2}, v_{1}^{\prime}$, and $v_{2}^{\prime}$. Eliminate the two terms involving $v_{1}^{\prime}$ and $v_{2}^{\prime}$ by setting their sum equal to zero. This leaves just two terms in $y_{p}^{\prime}$. Now compute $y_{p}^{\prime \prime}$.
(d) Plug $y_{p}=v_{1} y_{1}+v_{2} y_{2}$ and $y_{p}^{\prime \prime}$ from Part (c) back into the original ODE. Things should cancel, leaving you with an equation involving $v_{1}^{\prime}$ and $v_{2}^{\prime}$ (no $v_{1}$ or $v_{2}$ ).
(e) Write a system of two first order ODEs involving $v_{1}^{\prime}$ and $v_{2}^{\prime}$. One equation comes from Part (c) and the other from Part (d). Write this in matrix form as well.
(f) Solve for $v_{1}^{\prime}$ and $v_{2}^{\prime}$ by multiplying your two equations by suitible functions and and or subtracting them to eliminate the other.
(g) Integrate to find $v_{1}$ and $v_{2}$ (feel free to use a computer). Write the general solution to the original ODE.

