# Math 2080: Differential Equations Worksheet 3.8: Power series solutions 

## NAME:

In this problem, you will solve the 2 nd order linear ODE $y^{\prime \prime}+x^{2} y^{\prime \prime}-4 x y^{\prime}+6 y=0$ using the power series method.
(a) Assume that the solution has the form $y(x)=\sum_{n=0}^{\infty} a_{n} x^{n}$. Plug $y(x)$ back into the ODE.
(b) Shift the coefficients of the term containing $x^{n-2}$, and then factor out $x^{n}$ to get $\sum_{n=0}^{\infty}[\cdots] x^{n}$, and set $[\cdots]=0$ to get a recurrence relation for $a_{n+2}$ in terms of $a_{n}$.
(c) Explictly compute $a_{n}$ in terms of $a_{0}$ and $a_{1}$ for $n \leq 5$.
(d) How many distinct polynomial solutions are there?
(e) Write the general solution as $y(x)=a_{0} y_{0}(x)+a_{1} y_{1}(x)$, where $y_{0}$ and $y_{0}(x)$ and $y_{1}(x)$ are the even and odd terms respectively upon setting $a_{0}=a_{1}=1$.

