## Math 2080: Differential Equations Worksheet 4.3: Mixing with two tanks

## NAME:

Tank $A$ contains 10 gallons of a solution in which 5 oz of salt are dissolved. Tank $B$ contains 20 gallons in which 6 oz of salt are dissolved. Salt water with a concentration of $2 \mathrm{oz} / \mathrm{gal}$ flows into each tank at a rate of $4 \mathrm{gal} / \mathrm{min}$. The fully mixed solution drains from Tank $A$ at a rate of $3 \mathrm{gal} / \mathrm{min}$ and from Tank $B$ at a rate of $5 \mathrm{gal} / \mathrm{min}$. Solution flows from Tank $A$ to Tank $B$ at a rate of $1 \mathrm{gal} / \mathrm{min}$. Let $\mathbf{x}(t)=\left[\begin{array}{l}x_{1}(t) \\ x_{2}(t)\end{array}\right]$, where $x_{1}(t)$ (respectively, $x_{2}(t)$ ) is the amount of salt in Tank $A$ (resp., Tank $B$ ) after time $t$.
(a) Write down a system of ODEs (including the initial condition $\mathbf{x}(0))$ that models this situation, and write it in matrix form: $\mathbf{x}^{\prime}=\mathbf{A x}+\mathbf{b}, \mathbf{x}(0)=\mathbf{c}$.
(b) What is the steady-state solution, $\mathbf{x}_{s s}$ ?
(c) If $\mathbf{x}_{s s}=\left[\begin{array}{l}a \\ b\end{array}\right]$, then change variables by setting $y_{1}=x_{1}-a$ and $y_{2}=x_{2}-b$. Plug $y_{1}$ and $y_{2}$ back into the system to get a related system in $\boldsymbol{y}=\left[\begin{array}{l}y_{1} \\ y_{2}\end{array}\right]$. Don't forget the initial condition, $\boldsymbol{y}(0)$.

