

## Math 2080: Differential Equations

### Worksheet 4.4: Solving a $2 \times 2$ system of ODEs

NAME:

Consider the system of differential equations: 
$$\begin{cases} x_1' = -2x_1 + x_2 - 12, & x_1(0) = 5 \\ x_2' = 4x_1 + x_2 - 18, & x_2(0) = 20 \end{cases}$$

1. Write this in matrix form,  $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{b}$ , and find the steady-state solution,  $\mathbf{x}_{ss}$ .

2. Make a change of variables to transform the system into a homogeneous system.

3. Solve the homogeneous system. The eigenvalues of  $\mathbf{A}$  are  $\lambda_1 = -3$ ,  $\lambda_2 = 2$ , with eigenvectors  $\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$  and  $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ .

4. Solve the solution to the original system of differential equations, by writing  $\mathbf{x} = \mathbf{x}_h + \mathbf{x}_{ss}$ .

5. Sketch the phases portrait of the homogeneous system, and the inhomogeneous system. Also sketch the particular solutions satisfying the given initial conditions.