## Math 2080: Differential Equations Worksheet 4.6: Phase portraits with complex eigenvalues

NAME:

Consider the system of differential equations: $\begin{cases}x_{1}^{\prime}=3 x_{1}+2 x_{2}, & x_{1}(0)=0 \\ x_{2}^{\prime}=-5 x_{1}+x_{2}, & x_{2}(0)=1\end{cases}$
(a) Write this in matrix form, $\boldsymbol{x}^{\prime}=\boldsymbol{A} \boldsymbol{x}, \boldsymbol{x}(0)=\boldsymbol{x}_{0}$.
(b) Given that the eigenvalues of $\boldsymbol{A}$ are $\lambda_{1}=2+3 i$ and $\lambda_{2}=2-3 i$, with associated eigenvectors $\boldsymbol{v}_{1}=\left[\begin{array}{c}-1-3 i \\ 5\end{array}\right]$ and $\boldsymbol{v}_{2}=\left[\begin{array}{c}-1+3 i \\ 5\end{array}\right]$, write the general solution to $\boldsymbol{x}^{\prime}=\boldsymbol{A} \boldsymbol{x}$.
(c) Write the general solution as a linear combination of real-valued functions: $\boldsymbol{x}(t)=C_{1} \boldsymbol{u}(t)+C_{2} \boldsymbol{w}(t)$.
(d) Find the particular solution satisfying the initial condition.
(e) The phase portrait will consist of spiraling ellipses. To determine whether the spirals are clockwise ore counterclockwise, compute the $\boldsymbol{x}^{\prime}(0)=\left[\begin{array}{l}x_{1}^{\prime}(0) \\ x_{2}^{\prime}(0)\end{array}\right]$ and see which direction it points.
(f) Sketch the phase portrait of the system, and sketch the particular solution satisfying the initial condition. Feel free to use a compute to plot the approximate tilt and shape of the ellipse.

