## Math 2080: Differential Equations Worksheet 4.6: Phase portraits with complex eigenvalues

## NAME:

Consider the system of differential equations:  $\begin{cases} x_1' = 3x_1 + 2x_2, & x_1(0) = 0\\ x_2' = -5x_1 + x_2, & x_2(0) = 1 \end{cases}$ 

(a) Write this in matrix form,  $\mathbf{x}' = \mathbf{A}\mathbf{x}, \ \mathbf{x}(0) = \mathbf{x}_0.$ 

- (b) Given that the eigenvalues of  $\mathbf{A}$  are  $\lambda_1 = 2 + 3i$  and  $\lambda_2 = 2 3i$ , with associated eigenvectors  $\mathbf{v}_1 = \begin{bmatrix} -1 3i \\ 5 \end{bmatrix}$  and  $\mathbf{v}_2 = \begin{bmatrix} -1 + 3i \\ 5 \end{bmatrix}$ , write the general solution to  $\mathbf{x}' = \mathbf{A}\mathbf{x}$ .
- (c) Write the general solution as a linear combination of *real-valued* functions:  $\boldsymbol{x}(t) = C_1 \boldsymbol{u}(t) + C_2 \boldsymbol{w}(t)$ .

(d) Find the particular solution satisfying the initial condition.

(e) The phase portrait will consist of spiraling ellipses. To determine whether the spirals are clockwise ore counterclockwise, compute the  $\mathbf{x}'(0) = \begin{bmatrix} x'_1(0) \\ x'_2(0) \end{bmatrix}$  and see which direction it points.

(f) Sketch the phase portrait of the system, and sketch the particular solution satisfying the initial condition. Feel free to use a compute to plot the approximate tilt and shape of the ellipse.