Math 2080: Differential Equations
Worksheet 5.2: Properties & applications of Laplace transforms

NAME:

The following properties of the Laplace transform will be useful in this worksheet:

(i) $\mathcal{L}\{e^{at}\}(s) = \dfrac{1}{s-a}$

(ii) $\mathcal{L}\{t^n\}(s) = \dfrac{n!}{s^{n+1}}$,

(iii) $\mathcal{L}\{\cos bt\}(s) = \dfrac{s}{s^2+b^2}$.

(iv) $\mathcal{L}\{\sin bt\}(s) = \dfrac{b}{s^2+b^2}$.

(v) $\mathcal{L}\{e^{ct}\, f(t)\}(s) = F(s-c)$

(vi) $\mathcal{L}\{t^n\, f(t)\}(s) = (-1)^n \frac{d^n}{ds^n} F(s)$

(vii) $\mathcal{L}\{y''(t)\}(s) = s^2 Y(s) - sy(0) - y'(0)$

1. Compute the Laplace transform of te^{3t} two ways: using Properties (v) and (vi).

2. Compute the Laplace transform of $e^{2t}\cos 6t$.

3. Compute the inverse Laplace transform of $Y(s) = \dfrac{3}{2-6s}$. (Factor out -6)

4. Compute the inverse Laplace transform of $Y(s) = \dfrac{1}{(s-3)(s+1)}$. (Partial fractions)

5. Compute the inverse Laplace transform of $Y(s) = \dfrac{1}{s^2 + 4s + 13}$. (Complete the square)

6. Compute the inverse Laplace transform of $Y(s) = \dfrac{s}{s^2 + 4s + 13}$. (Complete the square)

7. In this problem, you will solve the initial value problem: $y'' - y = e^{2t}$, $y(0) = 1$, $y'(0) = 0$.

   (a) Take the Laplace transform of the initial value problem and solve for Y.

   (b) Use partial fraction decomposition to break up your equation for $Y(s)$.

   (c) Take the inverse Laplace transform of each fraction to get the solution to the initial value problem.