Math 2080: Differential Equations
Worksheet 5.3: Discontinuous forcing terms

NAME:

The following properties of the Laplace transform will be useful in this worksheet:

(i) $\mathcal{L}\{e^{at}\}(s) = \dfrac{1}{s-a}$

(ii) $\mathcal{L}\{t^n\}(s) = \dfrac{n!}{s^{n+1}}$,

(iii) $\mathcal{L}\{\sin bt\}(s) = \dfrac{b}{s^2+b^2}$.

(iv) $\mathcal{L}\{y''(t)\}(s) = s^2Y(s) - sy(0) - y'(0)$

(v) $\mathcal{L}\{f(t-c)\,H(t-c)\}(s) = e^{-cs}\,F(s)$

1. Compute $\mathcal{L}\{(t-2)^2\,H(t-2)\}(s)$.

2. Compute $\mathcal{L}\{t^2\,H(t-2)\}(s)$.

3. Compute $\mathcal{L}\{e^{t-3}\,H(t-3)\}(s)$.

4. Compute $\mathcal{L}\{e^{t+3}\,H(t-3)\}(s)$.

5. Consider the initial value problem $y''+y = f(t)$, $y(0) = 0$, $y'(0) = 1$, where $f(t) = \begin{cases} t, & 0 \le t \le 3 \\ 3, & t > 3 \end{cases}$

(a) Sketch $f(t)$, and write it using the Heavyside function.

(b) Take the Laplace transform of the differential equation, and solve for $Y(s)$.

(c) Use partial fractions to decompose $Y(s)$ into four terms. [Note: $\dfrac{1}{s^2(s^2+1)} = \dfrac{1}{s^2} - \dfrac{1}{s^2+1}$.]

(d) Apply the inverse Laplace transfrom to each term and write the solution to the IVP using the Heavyside function.

(e) Write the solution as a piecewise function (i.e., *not* using the Heavyside function).