Math 2080: Differential Equations
Worksheet 8.1: Modeling with nonlinear systems

NAME:

1. The following is called the *SIS model*:

$$S' = -\alpha SI + \gamma I$$
$$I' = \alpha SI - \gamma I$$

   (a) What disease might be modeled well by the SIS framework?

   (b) Find all steady-state solutions (S^*, I^*). Are these biologically reasonable? An steady-state for which $I^* > 0$ is called an *endemic equilibrium*. Can an SIS disease be endemic?

2. Recall the *SIR model* in epidemiology:

$$
\begin{aligned}
S' &= -\alpha SI + \gamma I \\
I' &= \alpha SI - \gamma I \\
R' &= \gamma R
\end{aligned}
$$

If $I'(0) > 0$, then the disease will take of and become an epidemic. However, if $I'(0) < 0$, then the disease will quickly die off. The *basic reproductive number* of disease is defined as $\mathcal{R}_0 := \frac{\alpha}{\gamma} S_0$.

(a) Using the observation that

$$\mathcal{R}_0 = \frac{\alpha}{\gamma} S_0 = (\alpha S_0)(\gamma^{-1})$$

with

- $\alpha S_0 = \#$ new infections per person per day,
- $\gamma^{-1} = \#$ average duration of infection,

describe what physical quantity $\mathcal{R}_0$ represents.

(b) Find the *threshold* T such that if $\mathcal{R}_0 > T$, then the outbreak becomes an epidemic, but if $\mathcal{R}_0 < T$, it dies off.