1. **(8 points)** `Library/UMass-Amherst/Abstract-Algebra/PS-Relations/Relations6.pg`

In this problem we work out step-by-step the procedure for checking an equivalence relation.

Denote by $\mathbb{Z}$ the set of all integers. Declare that two integers x, y are related if $x - y$ is an integer multiple of 5. In symbols:

$$x \sim y \iff 5 \text{ divides } x - y.$$

We want to check if this is an equivalence relation. That means we need to check if $\sim$ is
(1) Reflexive
(2) Symmetric
(3) Transitive

We begin with (1). This means checking to make sure that for all integers x, we have $x \sim x$. Recall the definition of $\sim$ for this problem and we see that this is equivalent to saying that for all integers x, we have that $(x - x)$ is an integer multiple of 5.

Is this true? If so, enter Y; if not, enter an integer for which this is false. _____

Next, we check (2). This means checking to make sure that for all integers x, y, we have $x \sim y \Leftrightarrow y \sim x$. Unwind the definition of $\sim$ as we have done for (1) and we see that

$x \sim y \iff$ _____ $= 5m$ for some integer m

$y \sim x \iff$ _____ $= 5m$ for some integer m

Based on that, is (2) true? If so, enter Y; if not, enter a pair of integers for which this is false.
_____

Finally, we check (3). This means checking to make sure that for all integers x, y, z, if $x \sim y$ and $y \sim z$ then $x \sim z$.

Is this true? If so, enter Y; if not, give a triple of integers for which this fails. _____

Finally, based on this calculation, is $\sim$ an equivalence relation on the set of integers? Enter Y or N. _____

2. **(8 points)** `Library/UMass-Amherst/Abstract-Algebra/PS-Relations/Relations2.pg`

For the following relations on the set of college students, determine if it satisfies each of the following conditions:

	Reflexive	Symmetric	Transitive	Equivalence Relation
$A \sim B \Leftrightarrow A$ is shorter than B	_____	_____	_____	_____
$A \sim B \Leftrightarrow A, B$ took 3 class(es) together	_____	_____	_____	_____
$A \sim B \Leftrightarrow A, B$ have the same major	_____	_____	_____	_____

Please enter Y or N in each of the boxes.

3. **(8 points)** `Library/UMass-Amherst/Abstract-Algebra/PS-Relations/Relations3.pg`

For the following relations on the set of POSITIVE integers, determine if it satisfies each of the following conditions:

	Reflexive	Symmetric	Transitive	Equivalence Relation
$m \sim n \Leftrightarrow 13$ divides $m-n$	_____	_____	_____	_____
$m \sim n \Leftrightarrow 19$ divides $m+n$	_____	_____	_____	_____
$m \sim n \Leftrightarrow 11$ divides mn	_____	_____	_____	_____

Please enter Y or N in each of the boxes.

4. **(8 points)** Library/UMass-Amherst/Abstract-Algebra/PS-Relations/Relations1.pg

For each of the following relations on the set of real numbers, determine if it satisfies each the following conditions (enter Y or N in each of the boxes):

	Reflexive	Symmetric	Transitive	Equivalence Relation
$x \sim y \Leftrightarrow x^2 > y^2$	_____	_____	_____	_____
$x \sim y \Leftrightarrow x \leq y$	_____	_____	_____	_____
$x \sim y \Leftrightarrow x^2 + y^2 = 9$	_____	_____	_____	_____
$x \sim y \Leftrightarrow \lvert x-y \rvert < 6$	_____	_____	_____	_____
$x \sim y \Leftrightarrow xy = 0$	_____	_____	_____	_____

5. **(8 points)** Library/UMass-Amherst/Abstract-Algebra/PS-Relations/Relations4.pg

For the following relations on the set of points on the plane, determine if it satisfies each of the following conditions (please enter Y or N in each of the boxes):

	Reflexive	Symmetric	Transitive	Equivalence Relation
$(u_1, u_2) \sim (w_1, w_2) \Leftrightarrow u_2 = w_2$	_____	_____	_____	______
$(x_1, y_1) \sim (x_2, y_2) \Leftrightarrow$ either $(x_1, y_1) = (x_2, y_2)$ or the line segment joining the two points have a slope > 15	_____	_____	_____	______
$(x_1, y_1) \sim (x_2, y_2) \Leftrightarrow$ the distance between the points is > 9	_____	_____	_____	______

6. **(8 points)** Library/MC/Proofs/Relations/Equvalence01.pg

Order 7 of the following sentences so that they form a logical proof of the statement:

Suppose R is a symmetric and transitive relation on A (i.e. $A \times A$) . Further suppose that for each $a \in A$ that there exists $b \in A$ such that $(a,b) \in R$.

Show: R is an equivalence relation.

Quick Hint? **What makes R an equivalence relation?**

- $(a,a) \in R \implies \exists b \in A$ such that $(a,b) \in R$ and $(b,a) \in R$ by transitivity
- $\exists b \in A$ such that $(a,b) \in R$
- Let (a,b) be an arbitrary element of R.
- R is reflexive

- $(a,b) \in R$ and $(b,a) \in R$ implies $(a,a) \in R$ by transitivity
- Assume R is symmetric and transitive and $\forall a \in A, (a,a) \in R$.
- $(b,a) \in R$ by symmetry
- Too much may be the equivalent of none at all.
- Let a be an arbitrary element of A.
- R is an equivalence relation
- Assume that R is symmetric and transitive on A and that each element in A is related to at least one other element in A.

7. **(8 points)** `Library/MC/Proofs/Relations/Equvalence02.pg`

Order 10 of the following sentences so that they form a logical proof of the statement:

For $A = Z \times Z$, define a relation R on A by:
$((a,b),(c,d)) \in R \iff ad = bc$
Prove that R is an equivalence relation on A.

- Hence, R is symmetric since $((a,b),(a,b)) \in R$.
- Hence R is symmetric. ¡br¿ Next consider $(a,b)R(c,d)$ and $(c,d)R(e,f)$
- Hence R is reflexive.
- Hence, R is transitive. ¡br¿ For any $(a,b), ab = ba$.
- $af = be \implies (a,b)R(e,f)$
- Define R on $Z \times Z$ such that $((a,b),(c,d)) \in R \iff ad = bc$
- Thus R is an equivalence relation.
- Then, $ad = bc$ and $cf = de$ and so $af = be$.
- Nathan is a goob.
- Then $ad = bc \implies bc = ad$ and so $(c,d)R(a,b)$.
- Hence, R is reflexive and $(a,b)R(c,d)$ means R is symmetric and transitive.
- Consider $((a,b),(c,d)) \in R$.
- $(a,b)R(c,d) \implies ab = cd$.
- Therefore $(a,b)R(a,b)$

8. **(8 points)** `Library/MC/Proofs/Relations/Partition02.pg`

Among the options below there are 7 different partitions of the set $A = \ 0,1,2,\ldots,21\ $. List them on the right according to the number of equivalence classes that each partition induces.

- 1,2,...,5,6,8,...,20
- 0,21,1,20,2,19,3,18,...,10,11
- $Z_0, Z_1, Z_2, Z_3, Z_4, Z_5$, ¡br¿ $Z_6, Z_7, \ldots, Z_{20}, Z_{21}$
- 0,1,2,...20,21
- even positive integers less than 21,¡br¿odd positive integers less than 21,0,21
- even positive integers less than 21,¡br¿odd positive integers less than 21,0,21
- 1,2,...20,21,22
- 0,1,2,...,21
- S_0 =0,S_1 =3,6,9,S_2 =1,4,7,10,S_3 =2,5,8,11,$A - S_0 - S_1 - S_2 - S_3$
- 0,1,2,...,8,9,10,...,21
- even numbers less than 21,¡br¿odd numbers less than 21,21

9. **(6 points)** `Library/UMass-Amherst/Abstract-Algebra/PS-Relations/Relations5.pg`

Determine all pairs of integers A,B such that
$(m,n) \sim (u,v) \iff m - An = u - Bv$

is an equivalence relation on the set of all pairs of integers.

$A =$ __________
$B =$ __________