

Department of Mathematical and Statistical Sciences
College of Science
Clemson University

MATH 4120-001, Modern Algebra

Spring 2022
MWF 9:05–9:55am
Bracket Hall 220

“This is the plan, up until it is no longer the plan.”
–Anonymous university administrator (not at CU)

Instructor: Matthew Macauley, Martin Hall O-325, macaule@clemson.edu, 656-1838 (no voicemail)

Course website: We will rarely use Canvas. All of the course material will be made freely available on the course website:

http://www.math.clemson.edu/~macaule/classes/s22_math4120/

Course Description: Group theory is the study of symmetry, and is one of the most beautiful areas in all of mathematics. It arises in puzzles, visual arts, music, nature, the physical and life sciences, computer science, cryptography, and of course, all throughout mathematics. This course will cover the basic concepts of group theory, with a special effort made to emphasize the intuition behind the concepts and motivate the subject matter. It will also cover basic Galois theory and ring theory, which are more advanced topics.

Many pictures and diagrams will be provided. We will analyze art freises, chemical molecules, wallpaper patterns, and braids. At the end of the semester, you will truly understand groups, rings, fields, subgroups, cosets, ideals, products and quotients, homomorphisms, group actions, conjugacy classes, centralizers, normalizers, semidirect products, theorems by Lagrange, Cayley, Cauchy, and Sylow, and what Évariste Galois stayed up until dawn writing the night before his untimely death in a duel at age 20, that remains one of the most celebrated achievements in all of mathematics.

In the end, you will leave with a new appreciation of the beauty (and difficulty) of an area of mathematics you never dreamt existed.

Prerequisite: Math 3110 (Linear Algebra) and Math 3190 (Introduction to Proofs).

Modality: This is an in-person class, but there is considerable uncertainty due to COVID, especially the omicron variant. We all must be prepared to shift to an online modality with little notice, and I will email a Zoom link that will be used if this happens. Though I plan to follow whatever the current university modality policy is, there is a strong chance that on at least one day, I will be sick (nearly impossible to avoid with a toddler at home), traveling due to a conference, or we will have a weather event like a hurricane, tornado, or snowstorm. If this happens, we will have class online (could be either synchronous or asynchronous), rather than it being canceled or taught by a substitute, and I will provide details ahead of time.

Communication Strategy: I prefer to communicate via emails rather than Canvas messages.

All of my email addresses (e.g., macaule@clemson.edu) go to the same gmail inbox, which I check multiple times a day. Though I have the gmail iPhone app, I don't get email push notifications. Also, I usually don't check email on Saturdays.

I will typically be in my office at least MWF, from roughly 8:00am–4:30pm. If you need to reach me immediately during the week, feel free to call my office phone, at (864) 656-1838. If I'm not there, please send an email rather than leaving a voicemail.

If you send me an email and do not get a reply by the time you go to bed, please re-send it, as that is my mistake. Just click "Reply" and "Send"; no need to explain.

Happy Hour: It has been my experience that evening Zoom office hours are much more frequently attended than (pre-COVID) on-campus office hours ever were. As such, three days a week at 7pm (days TBD), you are invited to join me and your classmates on Zoom for an Adult Beverage¹, company, and office hours. I'll stick around to answer questions as long as there are some. However, *I will only show up if at least one person RSVPs, either in class or by sending me an email before 6:30pm, saying they will attend.* If no one is there by 7:05pm, I will log off.

I am also available to meet by appointment, if desired, either over Zoom or in person. To make an appointment, email me and include block(s) of time in which you are available. Please let me know in advance if you want any Zoom meeting to be private, like if you want to discuss your grade. In that case, I will use a different Zoom meeting or a breakout room.

Textbooks: There does not yet exist a textbook that covers the material in this course how I will teach it. I am currently writing such a book, with a projected completion date of Summer 2022. My book was inspired by the delightfully wonderful 2009 book *Visual Group Theory*, by Nathan Carter. However, that was written for a more general audience, and so it is not rigorous enough for advanced math majors.

There is a freely available set of *Inquiry Based Learning* (IBL) lecture notes called *An Inquiry-Based Approach to Abstract Algebra*, by Dana C. Ernst. This follows the Visual Group Theory approach, and is available at

<http://danaernst.com/teaching/mat411f16/materials/>

in pdf form, both the entire e-book and the individual chapters. The IBL style means that the proofs are not included, as they are intended to be filled in by the students. However, it is still a very good reference.

The more traditional book *Abstract Algebra: Theory and Applications* by Tom Judson is also a good reference, and is freely available online.

A complete set of my lectures note slides will be posted on the course webpage. These, along with the book by Judson and the e-Book by Ernst, should more than suffice for resources.

Useful websites:

Course webpage: http://www.math.clemson.edu/~macaule/classes/s22_math4120/ (all relevant links posted here)

Canvas: <https://www.clemson.edu/canvas/> (will be used very sparingly, if at all)

Gradescope: <https://www.gradescope.com/> (click "Log in", then "School Credentials")

Homework: There will be 14 weekly homework assignments, usually consisting of 4–5 problems. Though it can be a difficult habit to form, *I strongly encourage students try to complete one problem per day, rather than save them all for the last 24–48 hours.*

¹For me, this means drinks like **LaCroix** or **Kombucha**, which are *very unpopular* among kids.

Working together is encouraged, but everyone must do their own work, and *collaborators must be cited*. Outside sources, such as webpages, must also be cited as well, if they were instrumental to a solution.

Homework assignments are due at 11:59pm EST, and will be submitted through Gradescope. Assignments can be submitted multiple times; only the last submission will be graded. There is a 3-hour grace period for deadlines, meaning that there is no penalty for assignments submitted within 3 hours of the deadline.

Assignments submitted after this period will not be accepted, but I will drop everybody's lowest score. This policy is intended to be an alternative to giving extensions due to unexpected circumstances such as an illness.

Any assignment that is typeset using L^AT_EX will get an automatic 24-hour extension.

Exams: There will be 2 in-class midterm exams, and one final exam. I will drop either your lowest midterm or one-half the final exam's weight. All exams will be closed notes and closed book.

Attendance: Please make an effort to attend all classes, and to be on time. I will try to show up at least 10 minutes early to all classes. In the unlikely scenario that I am absent 5 minutes after class has started, check your email. If you have not heard from me 10 minutes after class has begun, you may assume that class has been canceled.

If you cannot make class, due to illness, quarantine, or some other reason, please let me know as a courtesy.

Technology in class: One particular study on multitasking showed that students on laptops score 11% lower than those not on laptops. Moreover, students who were not on laptops but had a laptop in their "line of sight" scored *17% lower!* Therefore, the use of laptops and cell phone in class will not be allowed. Tablets may be used *only* for taking notes.

Exams: There will be two 50-minute midterm exams during the semester and a cumulative 2.5-hour final exam. If these are given in person (the default plan), then students must provide their own blank paper. Most likely, I will have students scan their completed exam with a smart phone app (e.g., CamScanner or AdobeScan) and submit the file on Canvas or Gradescope.

If any exams are taken online, then I will proctor them over Zoom. You must provide consent to having the meeting recorded.

Exam checklist (things to bring, regardless of modality):

- Plenty of blank scratch paper and pens or pencils.
- Smartphone (for scanning your exam when you finish).

Exam rules (if online):

- Before beginning the exam, you must do a "room scan" with your camera, and also verify that all of the paper you brought is indeed blank.
- You must share your video for the entire duration of the exam.
- The camera must be far enough away so I can see your hands and paper at all times. That is, I must be able to verify that you are not using a phone or computer.

- When you are finished, send me a private Zoom Chat to let me know, and then scan and email your exam to me while still on camera. It must be scanned in one multi-page pdf document, and *not* multiple individual one-page documents.

Before *and* after submitting, double-check to make sure that the scanned file is (i) fully legible, (ii) complete (iii) the correct file, and (iv) correctly oriented (not upside down or rotated).

It is strongly recommended that you practice with your smartphone scanning app *before* the exam.

Grading: Your final grade will be computed as follows:

| | |
|-----------------------|-----|
| Homework | 25% |
| Midterm 1 | 25% |
| Midterm 2 | 25% |
| Cumulative Final Exam | 50% |

I will drop either your lowest midterm grade, OR half the weight of your final exam; whichever is lowest. I also have the following “final exam policies,” though *they only apply if you have a passing grade on the homework*: If you get an unambiguous **A** on the final exam, then you will get an **A** in the course. If you get an unambiguous **B** or better on the final exam, then you will get (at least) a **B** in the course.

I do not impose arbitrary numeric cutoff lines for final grades, e.g., A=90+, B=80–89, etc. Rather, I grade by natural “clusters.” I tend to give difficult assignments and exams, and so the letter grades end up corresponding to lower numeric averages than they do in most classes.

Make-Up Policy: No make-up exams will be given. I will drop your lowest midterm, which means that if you miss a midterm, then your final exam grade will replace it. The homework deadlines will not be extended for individual students, and assigned homework must be turned in by the deadline.

Student Learning Outcomes: Upon successful completion of the course, students will be able to

Demonstrate a solid understanding of modern algebra (group theory, ring theory, and Galois theory) at the undergraduate level.

Explain to a friend or family member who knows nothing about mathematics what group theory is, how it arises, and why it’s beautiful.

Explain how group theory can be thought of as the study of symmetry, and how it arises in puzzles, visual arts, the sciences, and other branches of mathematics.

Use visual diagrams and pictures to demonstrate the important definitions and concepts of group theory (of course, in addition to being able to define them rigorously).

Learn to understand, read, and write rigorous mathematical arguments on topics in algebra.

Develop good mathematical writing skills. Important aspects of this are *accuracy*, *clarity*, and *conciseness*.

Key Dates

| | |
|-----------------|---|
| Jan 12 (Wed) | Classes begin |
| Jan 17 (Tue) | Martin Luther King Jr. holiday |
| Jan 19 (Tue) | Last day to register or add a class |
| Mar 18 (Fri) | Last day to drop a class or withdraw from the University w/o final grades |
| Mar 21–25 (W–F) | Spring break |
| Apr 29 (Fri) | Last day of class |
| May 6 (Fri) | Final Exam, 8–10:30am |

“No exceptions”: In any class syllabus, no matter how they are worded, policies and phrases like “no exceptions”, “no make-ups”, etc. are *never* actually what they sound, and this is especially true this semester. Things happen, from natural disasters (hurricanes, tornados), to human disasters (9/11, school shootings), to personal and family tragedies, to health emergencies (COVID, auto accidents, hospitalizations). This does not mean that any exception or extension will be granted, but I will do my best to be reasonable, fair, and accommodating.

Make-Up Policy: I will drop your lowest midterm, which means that if you miss a midterm, then your final exam grade will replace it. The homework deadlines will not be extended for individual students, and assigned homework must be turned in by the deadline. **PLAN AHEAD:** If you submit assignments minutes before the deadline, you take the risk of bad luck, e.g., a power outage, computer freeze or crash, personal emergency, zombie attack, etc., that could make you miss the deadline.

By default, any exam that was scheduled at the time of a class cancellation due to power outage / inclement weather will be given at the next class meeting. Any extension or postponement of assignments or exams must be granted by me via email or Canvas within 24 hours of the weather-related cancellation.

Mental health: Your mental health is important to me, and I am always available to talk. Please don’t hesitate to reach out. We’re in this together, and all of us are struggling in some regards, myself included.

Social media: If you want to connect with me on Social Media, then use LinkedIn or Twitter. I don’t really use the first one, though I have an account. I use the second one to promote my mathematical materials and current book project. I will not accept friend requests on Facebook, there is just too much potential for risk and liability.

Academic Integrity: “As members of the Clemson University community, we have inherited Thomas Green Clemson’s vision of this institution as a ‘high seminary of learning’. Fundamental to this vision is a mutual commitment to truthfulness, honor, and responsibility, without which we cannot earn the trust and respect of others. Furthermore, we recognize that academic dishonesty detracts from the value of a Clemson degree. Therefore, we shall not tolerate lying, cheating, or stealing in any form.”

Special Accommodations: Students with disabilities who need accommodations should make an appointment with Dr. Arlene Stewart, Director of Disability Services, to discuss specific needs within the first week of classes. Students should present a Faculty Accommodation Letter from Student Disability Services when they meet with instructors. Student Disability Services is located in Suite 239 Academic Success Building (656-6848; sds-l@clemson.edu). Please be aware that accommodations are not retroactive and new Faculty Accommodation Letters must be presented each semester.

Title IX Policy: Clemson University is committed to a policy of equal opportunity for all persons and does not discriminate on the basis of race, color, religion, sex, sexual orientation, gender, pregnancy, national origin, age, disability, veteran's status, genetic information or protected activity (e.g., opposition to prohibited discrimination or participation in any complaint process, etc.) in employment, educational programs and activities, admissions and financial aid. This includes a prohibition against sexual harassment and sexual violence as mandated by Title IX of the Education Amendments of 1972.

The University is committed to combatting sexual discrimination including sexual harassment and sexual violence. As a result, you should know that University faculty and staff members who work directly with students are required to report any instances of sexual harassment and sexual violence, to the University's Title IX Coordinator. What this means is that as your professor, I am required to report any incidents of sexual harassment, sexual violence or misconduct, stalking, domestic and/or relationship violence that are directly reported to me, or of which I am somehow made aware.

There are two important exceptions to this requirement about which you should be aware:

Confidential Resources and facilitators of sexual awareness programs such as "Take Back the Night and Aspire to be Well" when acting in those capacities, are not required to report incidents of sexual discrimination.

Another important exception to the reporting requirement exists for academic work. Disclosures about sexual harassment, sexual violence, stalking, domestic and/or relationship violence that are shared as part of an academic project, a research project, classroom discussion, or course assignment, are not required to be disclosed to the University's Title IX Coordinator.

This policy is at <http://www.clemson.edu/campus-life/campus-services/access/title-ix/>. Alesia Smith is the Executive Director for Equity Compliance and the Title IX Coordinator. Her office is at 223 Holtzendorff Hall, phone number is 864.656.3181, and email address is alesias@clemson.edu.

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