Lecture 3.3: Solving differential equations with Fourier series

Matthew Macauley

Department of Mathematical Sciences
Clemson University
http://www.math.clemson.edu/~macaule/

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Motivation

Recall the method of undetermined coefficients to solve a 2nd order linear inhomogeneous ODE $y'' + a(x)y' + b(x)y = f(x)$:

1. Solve the related homogeneous equation: $y_h'' + a(x)y_h' + b(x)y_h = 0$.
2. Guess the form of a particular solution $y_p(x)$.
3. Add these together: $y(x) = y_h(x) + y_p(x)$.

$f(x)$	guess
e^{kx}	$y_p(x) = ae^{kx}$
$c_k x^k + \cdots + c_1 x + c_0$	$y_p(x) = a_k x^k + \cdots + a_1 x + a_0$
$\sin kx$ or $\cos kx$	$y_p(x) = a\cos kx + b\sin kx$.

Question

What if the forcing term is a piecewise function like a square wave?

$f(x)$	guess
square wave	$y_p(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L}$

This is generally *much* easier than using Laplace transforms!

Example 1

Solve $y'' + 3y' + 2y = f(x)$, for the square wave of period 2: $f(x) = \begin{cases} 1 & 0 < x < 1 \\ -1 & -1 < x < 0 \end{cases}$

Example 2

Solve $y'' + \omega^2 y = f(x)$, $\omega \neq n\pi$, for the square wave of period 2: $f(x) = \begin{cases} 1 & 0 < x < 1 \\ -1 & -1 < x < 0 \end{cases}$