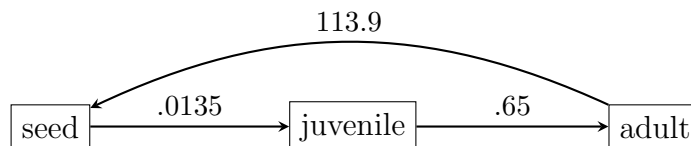


1. The life stages, transitions, and reproduct data for the South American annual *Hyptis suaveolens* (Schwarzkopf et. al, 2009) is shown below. Seeds germinate in late spring to early summer, and grow until the end of the wet season in December–January, when they produce seeds and die.



- (a) Build a population matrix A that describes this.
- (b) Is there the smallest power k of A such that every entry of A^k is positive? (If so, what is k ?) The existence of such a k means that A is *primitive*.
- (c) If a matrix is primitive, then the Perron-Frobenius theorem guarantees a unique largest positive eigenvalue, called its *dominant eigenvalue*. Find the dominant eigenvalue of A .
- (d) Discuss the long-term behavior of this model, and the intrinsic growth rate.
- (e) How does this answer change if seeds can remain viable for more than one year?
2. Consider a structured population model with matrix $P = \begin{bmatrix} .3 & 2 \\ .4 & 0 \end{bmatrix}$ (called a *Leslie matrix*):
- (a) By thinking about the biological meaning of each entry in this matrix, do you think it describes a growing or declining population. Would you guess the population size would change rapidly or slowly? Explain your reasoning.
- (b) Compute the eigenvalues and eigenvectors of the model. (Use a computer.)
- (c) What is the intrinsic growth rate?
- (d) Express the initial vector $\mathbf{x}_0 = (5, 5)$ as a sum of the eigenvectors.
- (e) Use your answer in the previous part to give a formula for the population vector \mathbf{x}_t .
- (f) What is the long-term behavior, $\lim_{t \rightarrow \infty} \mathbf{x}_t$?
3. A model given in (Cullen, 1985), based on data collected in (Nellis and Keith, 1976), describes a certain coyote population. The population is stratified in three classes: pup, yearling, and adult, and the matrix

$$P = \begin{bmatrix} .11 & .15 & .15 \\ .3 & 0 & 0 \\ 0 & .6 & .6 \end{bmatrix}$$

describes changes over a time step of 1 year.

- (a) Carefully explain what each entry in this matrix is saying about the population.
- (b) Find the intrinsic growth rate (dominant eigenvalue) and corresponding eigenvector. Feel free to use a computer.
- (c) Will the population grow or decline? Quickly or slowly?