1. Imagine a predator–prey interaction in which a certain number of the prey population cannot be eaten because of a refuge in their environment that the predator cannot enter. Give a real-life example two populations that might exhibit this feature, and propose a model of this. Either an ODE of difference equation framework is fine.

2. Consider the following model of two species:

$$\frac{dX}{dt} = r_1 X \left(1 - \frac{X}{M_1 + b_{12} Y}\right)$$
$$\frac{dY}{dt} = r_1 Y \left(1 - \frac{Y}{M_2 + b_{21} X}\right).$$

   (a) Describe what interactive behavior between species X and Y is implied by the model, and an example of two species that this might model. Include a discussion of what the parameters might represent.

   (b) Find the nullclines, $X' = 0$ and $Y' = 0$, sketch them on the XY-plane.

   (c) Find the steady states of this model, and determine their stability by linearization.

   (d) Summarize the ecological implications of your results.

3. In class, we saw that the model $\begin{cases} P_{t+1} = P_t(1 + 1.3(1 - P_t)) - .5P_tQ_t \\ Q_{t+1} = .3Q_t + 1.6P_tQ_t \end{cases}$

   has a steady-state equilibrium that is approached through oscillations. Because the discrete logistic model $P_{t+1} = P_t(1 + 1.3(1 - P_t))$ on which it is based has $r = 1.3$, we know that it alone would produced underdamped dynamics (=damped oscillations) rather than the overdamped dynamics that arise when $r < 1$. Thus, it is not clear whether the oscillations in the model above are inherient to the model or, simply due to $r > 1$.

   Explore this using the MATLAB program `twopop` with a number of values of r – less than and greater than 1.3 in the predator–prey model. Can you find a value or $r < 1$ that yields oscillations in the predator–prey model? If so, can you find a value of r that yields no oscillations, and where is the "threshold" between these two dynamical regimes? Are there any other "thresholds" where the qualitative dynamics changes? Include print-outs for a few different values of r.