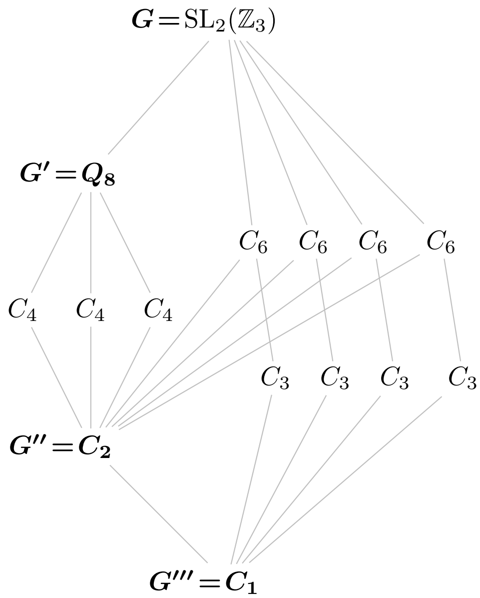
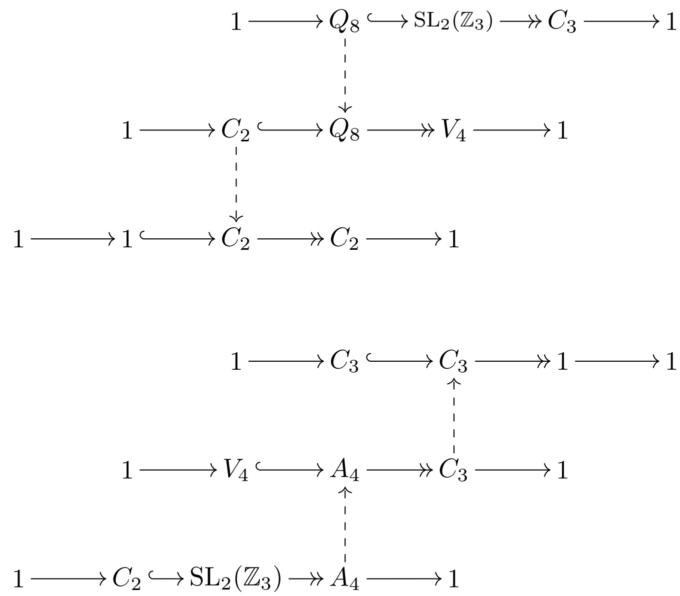
1. The group $G = \mathrm{SL}_2(\mathbb{Z}_3)$ is solvable with derived length 3. This means that it can be constructed as a sequence of abelian extensions. Moreover, this can be done two ways:
   - From "top-to-bottom": $G = G_0 \trianglerighteq G_1 \trianglerighteq G_2 \trianglerighteq G_3 = \langle 1 \rangle$.
   - From "bottom-to-top": $\langle 1 \rangle = G_3 \trianglelefteq G_2 \trianglelefteq G_1 \trianglelefteq G_0 = G$.

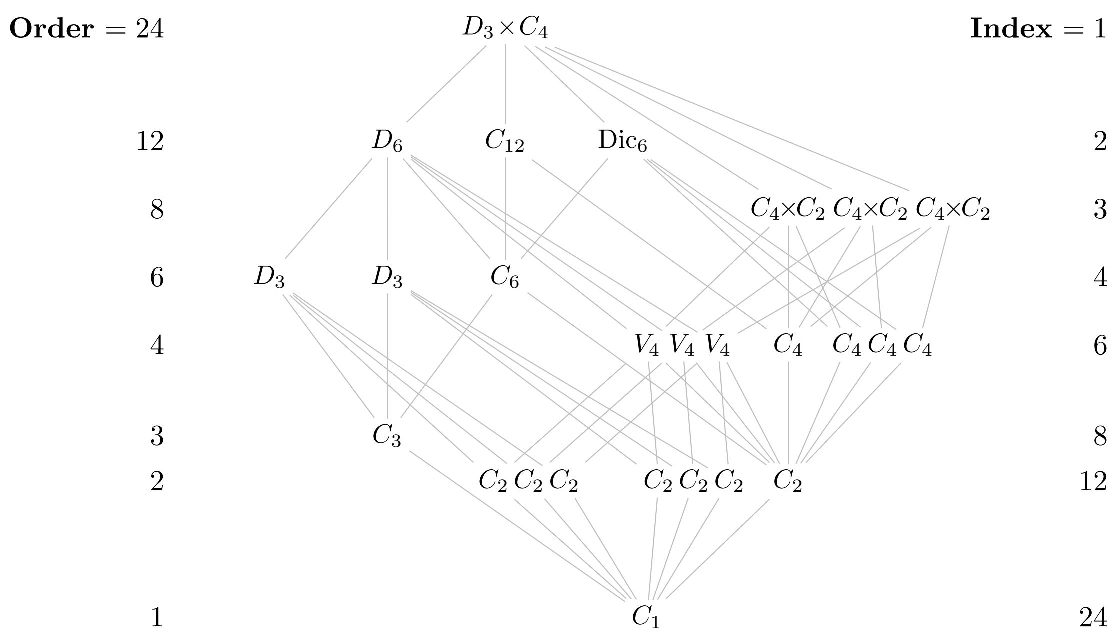
   These extensions can be encoded by short exact sequences, as shown below.

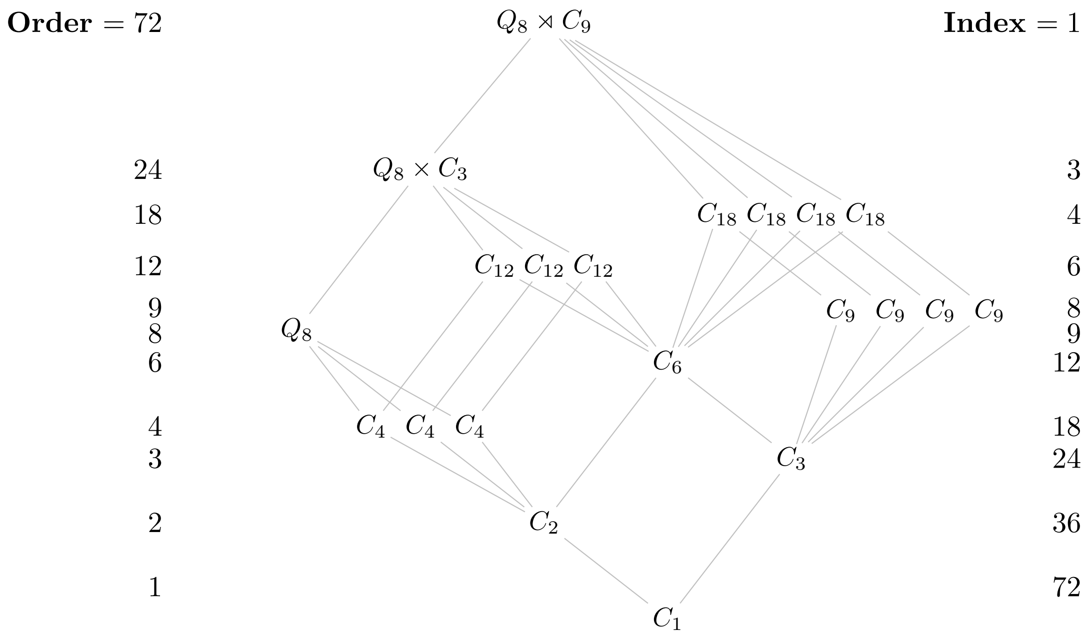




   Carry out this construction for the symmetric group $G = S_4$, which has derived length 3.

2. For both of the following groups G shown below, find all compositions series (up to isomorphism) and the composition factors. Then repeat the previous exercise, and find the abelianization of G.





3. Given a group G, prove the following elementary properties about its commutator subgroup, $G' = \langle xyx^{-1}y^{-1} \mid x, y \in G \rangle$.

   (a) $G' \trianglelefteq G$ and G/G' is abelian.

   (b) If $G' \leq H \leq G$, then $H \trianglelefteq G$.

   (c) If $N \trianglelefteq G$, then $N' \trianglelefteq G$.

   (d) If $N \trianglelefteq G$, then G/N is abelian if and only if $G' \trianglelefteq N$.

   (e) If $\phi \colon G_1 \to G_2$ is a homomorphism, then

   $$\phi([h,k]) = [\phi(h), \phi(k)] \qquad \text{and} \qquad \phi([H,K]) = [\phi(H), \phi(K)],$$

   for all $h, k \in G_1$, and $H, K \leq G_1$.