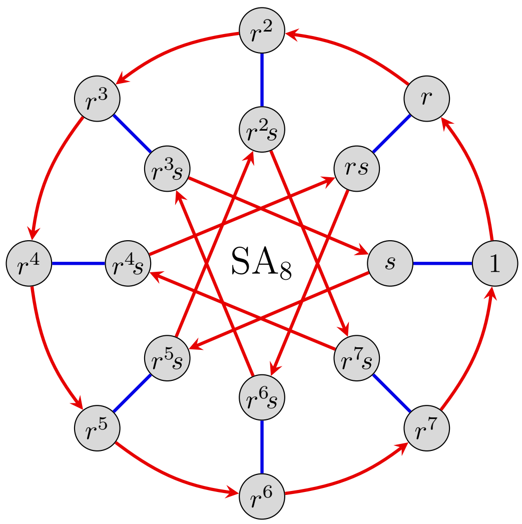
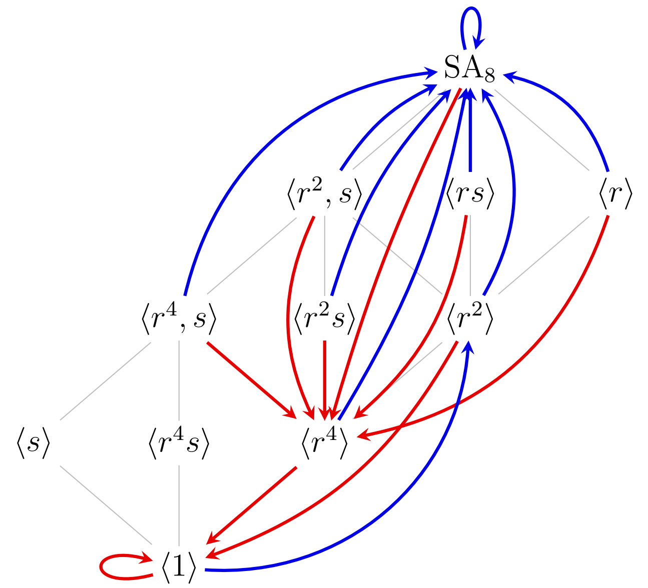
1. The *chutes and ladders diagram* of a finite group G is constructed by taking its subgroup lattice, and adding:

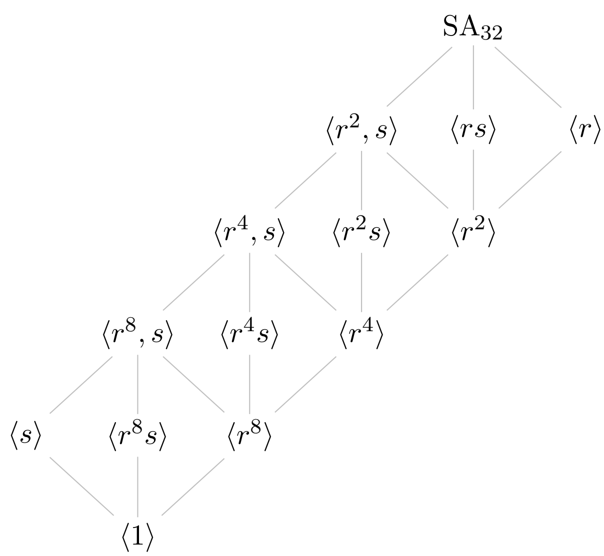
   - a red arrow for each "maximal central descent" $N \searrow L$, where $L = [G, N]$,
   - a blue arrow for each "maximal central ascent", $N \nearrow Z$, where $Z/N = Z(G/N)$.

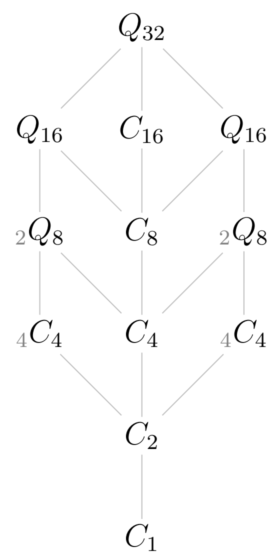
   An example is shown below for the semiabelian group $\mathrm{SA}_8 = \langle r, s \mid r^8 = s^2 = 1,\ srs = r^5 \rangle$.

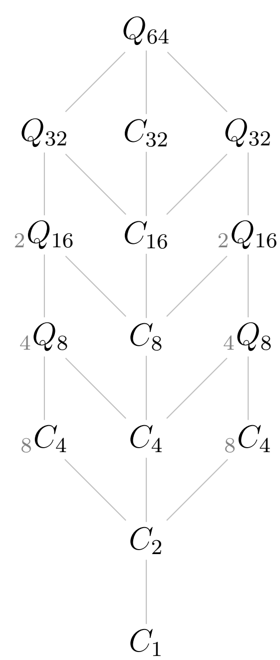




   (a) Construct the chutes and ladders diagram of the following groups: $D_3 \times C_4$ and $Q_8 \rtimes C_9$ (see previous HW for the lattices), and SA_{32}, Q_{32}, and Q_{64}.

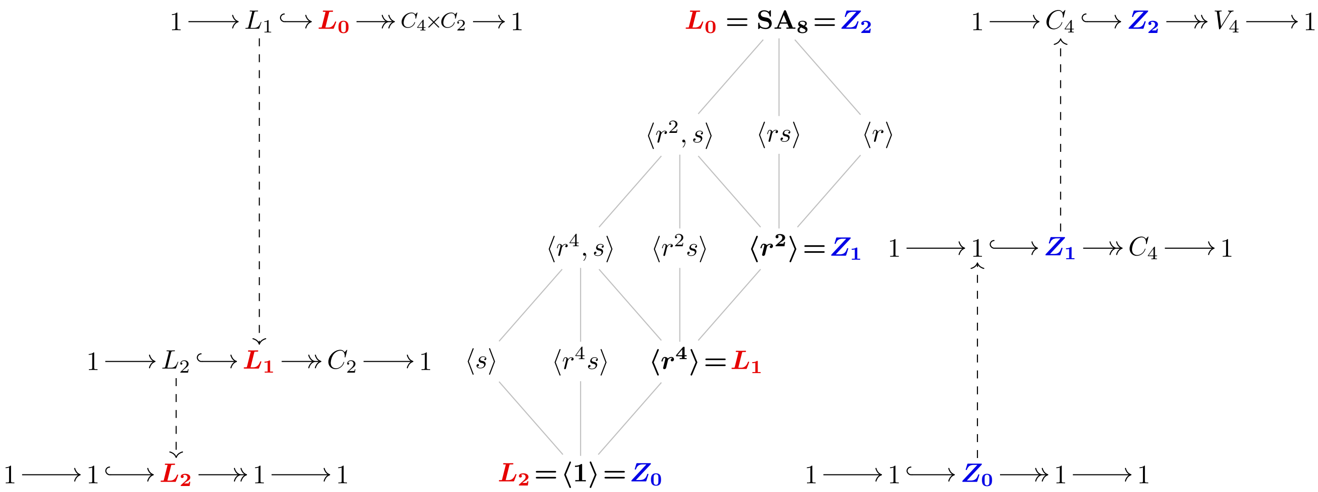




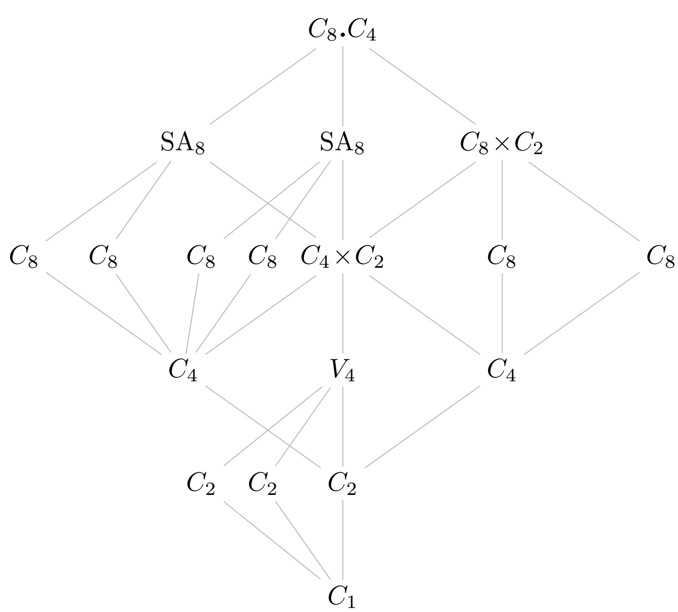


   (b) On the subgroups lattices of the groups from Part (a), mark the ascending and descending central series.

2. The *nilpotency class* of a group is the minimal number of steps it takes the ascending central series to reach the top of the subgroup lattice, or equivalently, the descending central series to reach the bottom. The following is an example of these for the semiabelian group SA_8, and the corresponding extensions.



(a) Construct the analogous picture for the nonsplit extension of C_8 by C_4, whose subgroup lattice is shown below. This is a minimal group that has nilpotency class 3, and whose ascending and descending central series do not coincide.



(b) Find the derived series $C_8.C_4 = G^{(0)} \trianglerighteq G' \trianglerighteq \cdots \trianglerighteq G^{(m)} = \langle 1\rangle$, and construct a chain of exact sequences, where the commutator subgroups appear as the middle terms.

3. Let G be a group with nilpotency class $n < \infty$.

(a) Show that every subgroup of G has nilpotency class at most n.

(b) Show that every homomorphic image of G has nilpotency class at most n.

(c) Show that nontrivial normal subgroups of G intersect the center nontrivially.