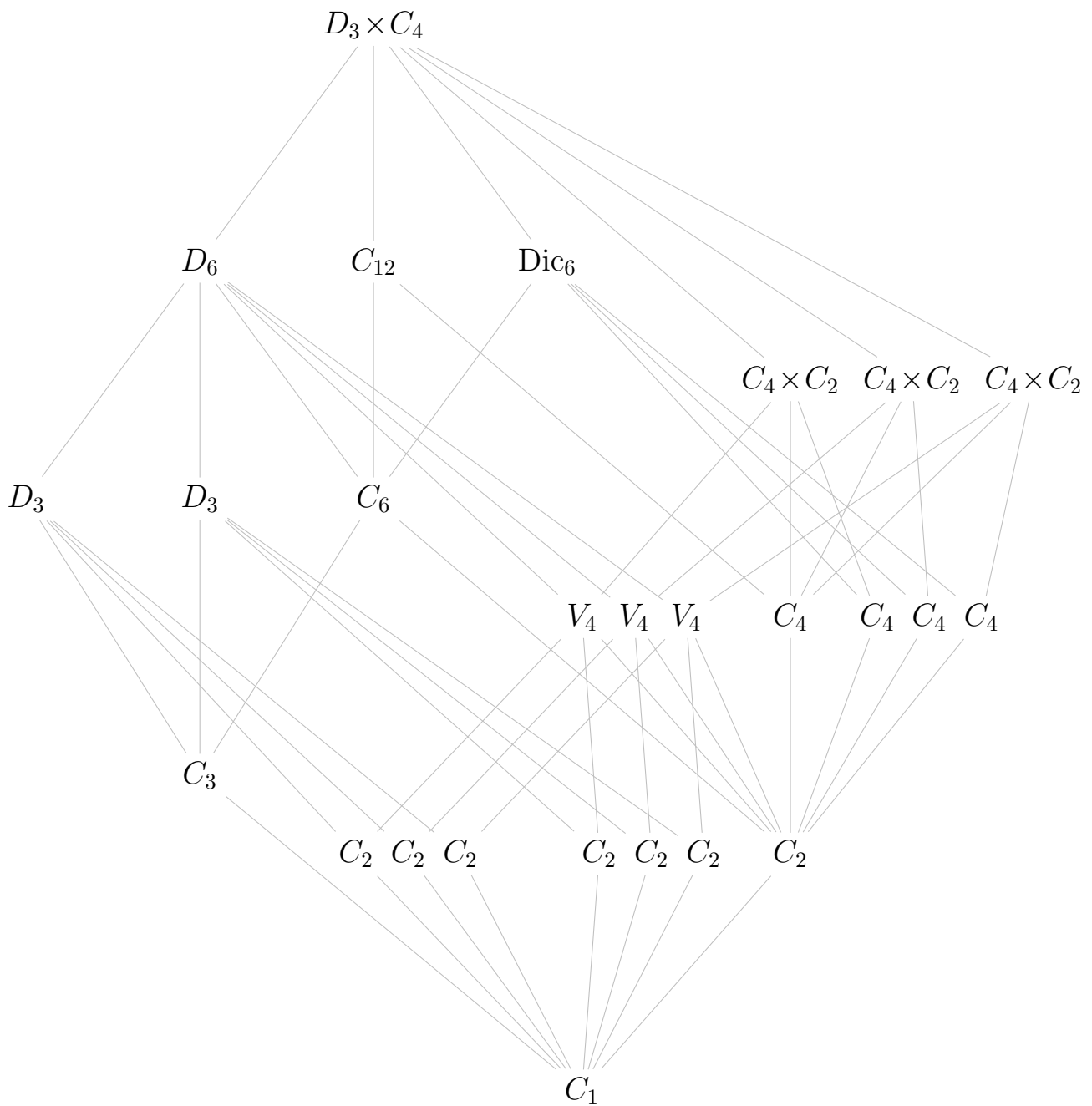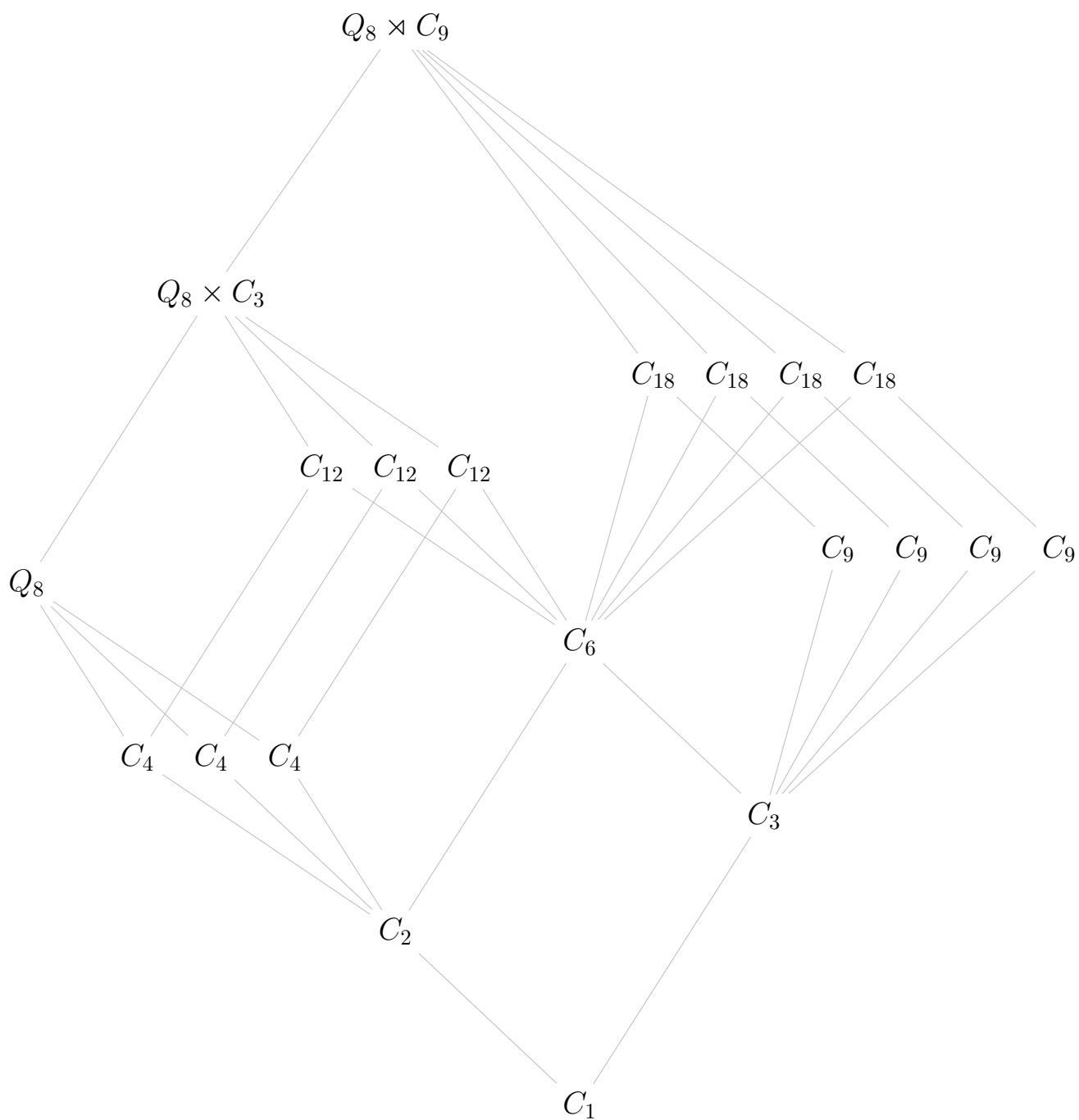# Supplemental material for Math 4130, HW 7

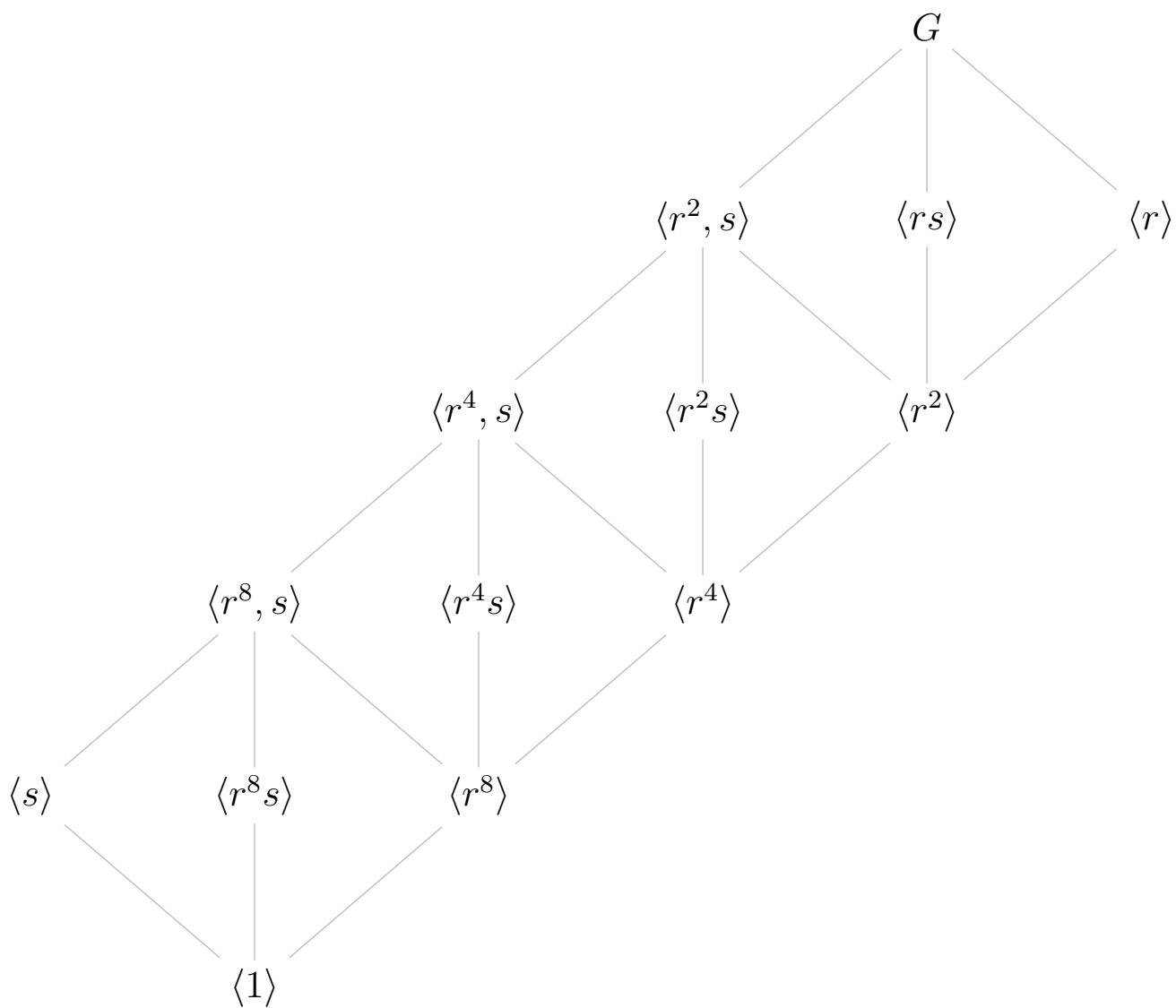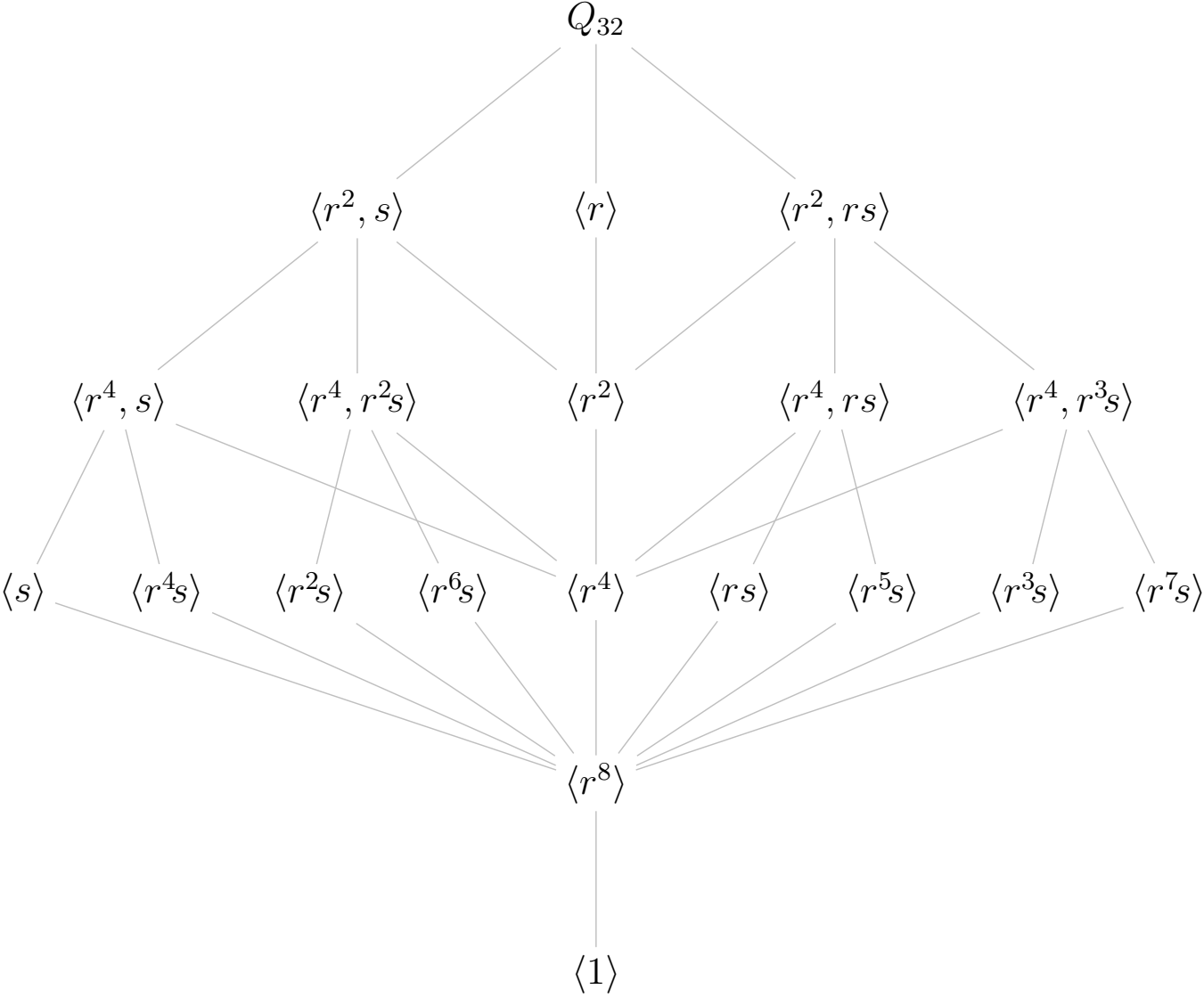**#1**: The chutes and ladders diagram of the group $D_3 \times C_4$, with the ascending and descending series labeled.

#1: The chutes and ladders diagram of the group $Q_3 \rtimes C_9$, with the ascending and descending series labeled.

$Q_8 \rtimes C_9$

$Q_8 \times C_3$

C_{18} C_{18} C_{18} C_{18}

C_{12} C_{12} C_{12}

C_9 C_9 C_9 C_9

Q_8

C_6

C_4 C_4 C_4

C_3

C_2

C_1

#1: The chutes and ladders diagram of the semiabelian group SA_{16}, with the ascending and descending series labeled.

G

$\langle r^2, s \rangle$ $\langle rs \rangle$ $\langle r \rangle$

$\langle r^4, s \rangle$ $\langle r^2 s \rangle$ $\langle r^2 \rangle$

$\langle r^8, s \rangle$ $\langle r^4 s \rangle$ $\langle r^4 \rangle$

$\langle s \rangle$ $\langle r^8 s \rangle$ $\langle r^8 \rangle$

$\langle 1 \rangle$

#1: The chutes and ladders diagram of the generalized quaternion group Q_{32}, with the ascending and descending series labeled.

Q_{32}

$\langle r^2, s\rangle$ $\langle r\rangle$ $\langle r^2, rs\rangle$

$\langle r^4, s\rangle$ $\langle r^4, r^2s\rangle$ $\langle r^2\rangle$ $\langle r^4, rs\rangle$ $\langle r^4, r^3s\rangle$

$\langle s\rangle$ $\langle r^4s\rangle$ $\langle r^2s\rangle$ $\langle r^6s\rangle$ $\langle r^4\rangle$ $\langle rs\rangle$ $\langle r^5s\rangle$ $\langle r^3s\rangle$ $\langle r^7s\rangle$

$\langle r^8\rangle$

$\langle 1\rangle$

#1: The chutes and ladders diagram of the generalized quaternion group Q_{64}, with the ascending and descending series labeled.

$$Q_{64}$$

$$Q_{32} \qquad C_{32} \qquad Q_{32}$$

$${}_2Q_{16} \qquad C_{16} \qquad {}_2Q_{16}$$

$${}_4Q_8 \qquad C_8 \qquad {}_4Q_8$$

$${}_8C_4 \qquad C_4 \qquad {}_8C_4$$

$$C_2$$

$$C_1$$
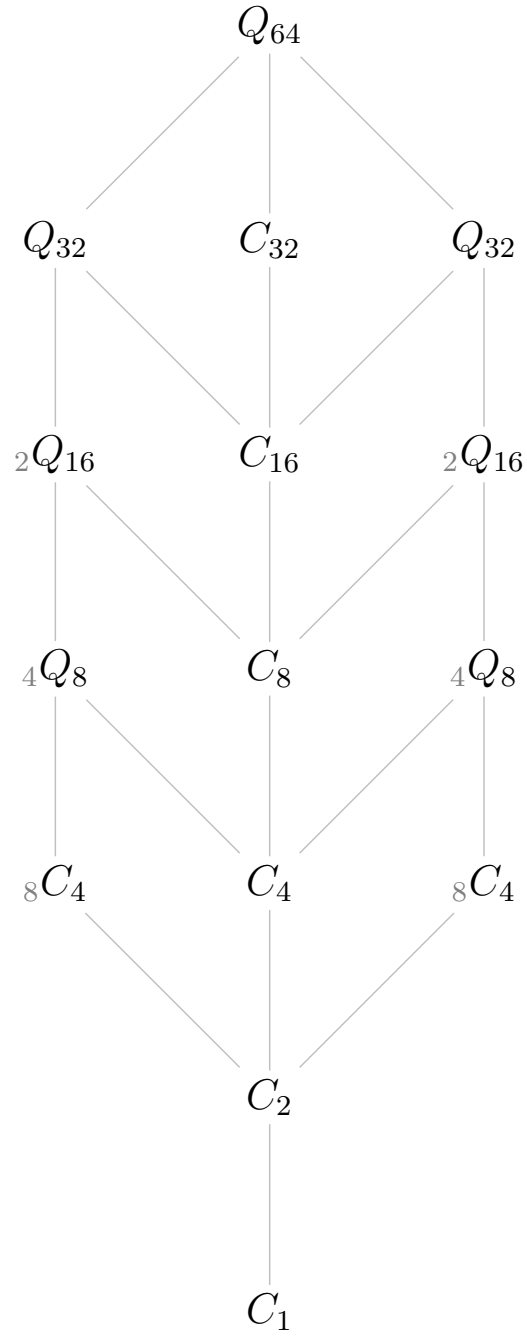
#2(a): The of ascending and descending series of the non-split extension $C_8.C_4$, with short exact sequences illustrating how to construct it as a series of central extensions.

$C_8.C_4$

SA$_8$ SA$_8$ $C_8 \times C_2$

C_8 C_8 C_8 C_8 $C_4 \times C_2$ C_8 C_8

C_4 V_4 C_2

C_2 C_2 C_2

C_1
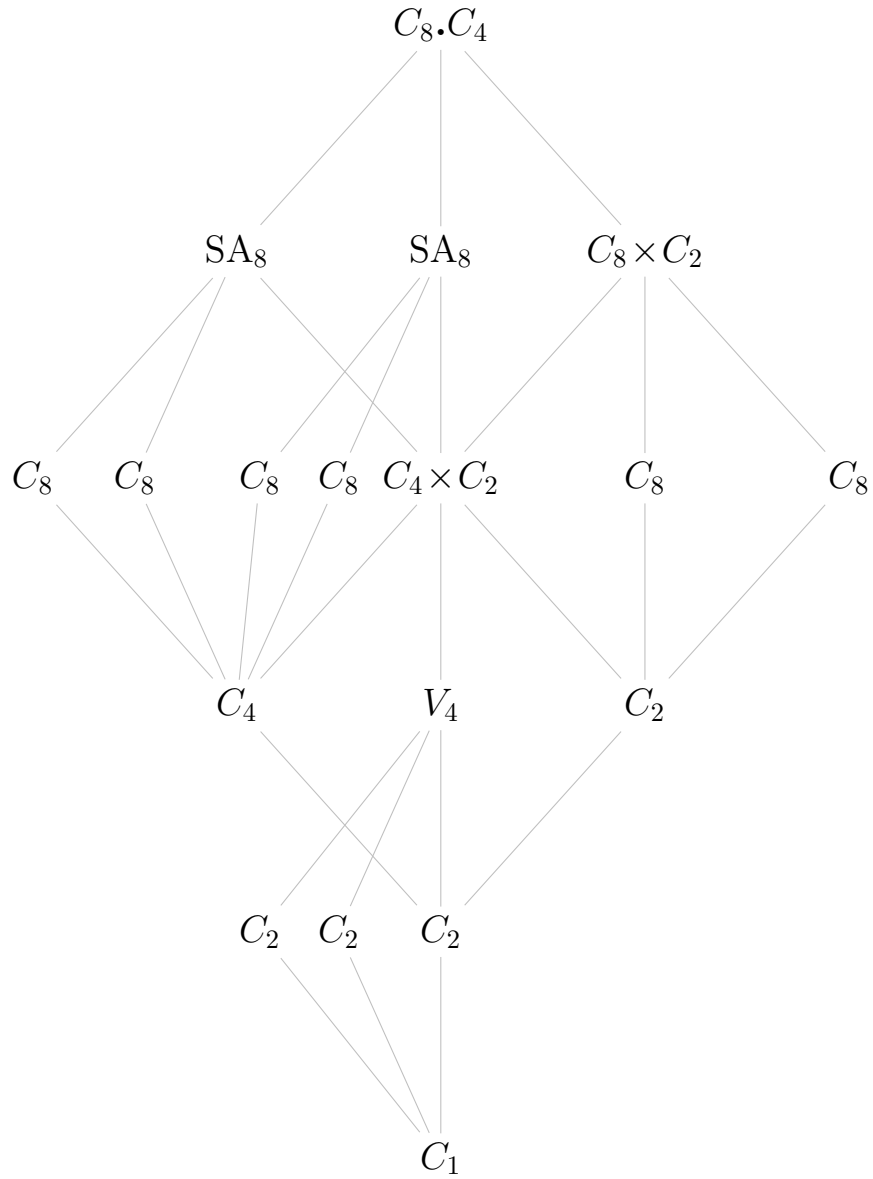
#2(b): The derived series $C_8.C_4 = G^{(0)} \trianglerighteq G' \trianglerighteq \cdots \trianglerighteq G^{(m)} = \langle 1 \rangle$, and a chain of exact sequences, where the commutator subgroups appear as the middle terms.

$$C_8.C_4$$

$$\text{SA}_8 \qquad \text{SA}_8 \qquad C_8 \times C_2$$

$$C_8 \quad C_8 \qquad C_8 \quad C_8 \quad C_4 \times C_2 \qquad C_8 \qquad C_8$$

$$C_4 \qquad V_4 \qquad C_2$$

$$C_2 \quad C_2 \qquad C_2$$

$$C_1$$