## Supplemental material for Math 4130, HW 9

**#1**: The subfield lattice of $\mathbb{Q}(\sqrt[8]{2}, \zeta) = \mathbb{Q}(\sqrt[8]{2}, i)$, the splitting field of $f(x) = x^8 - 2$, where $\zeta = e^{2\pi i/8}$.

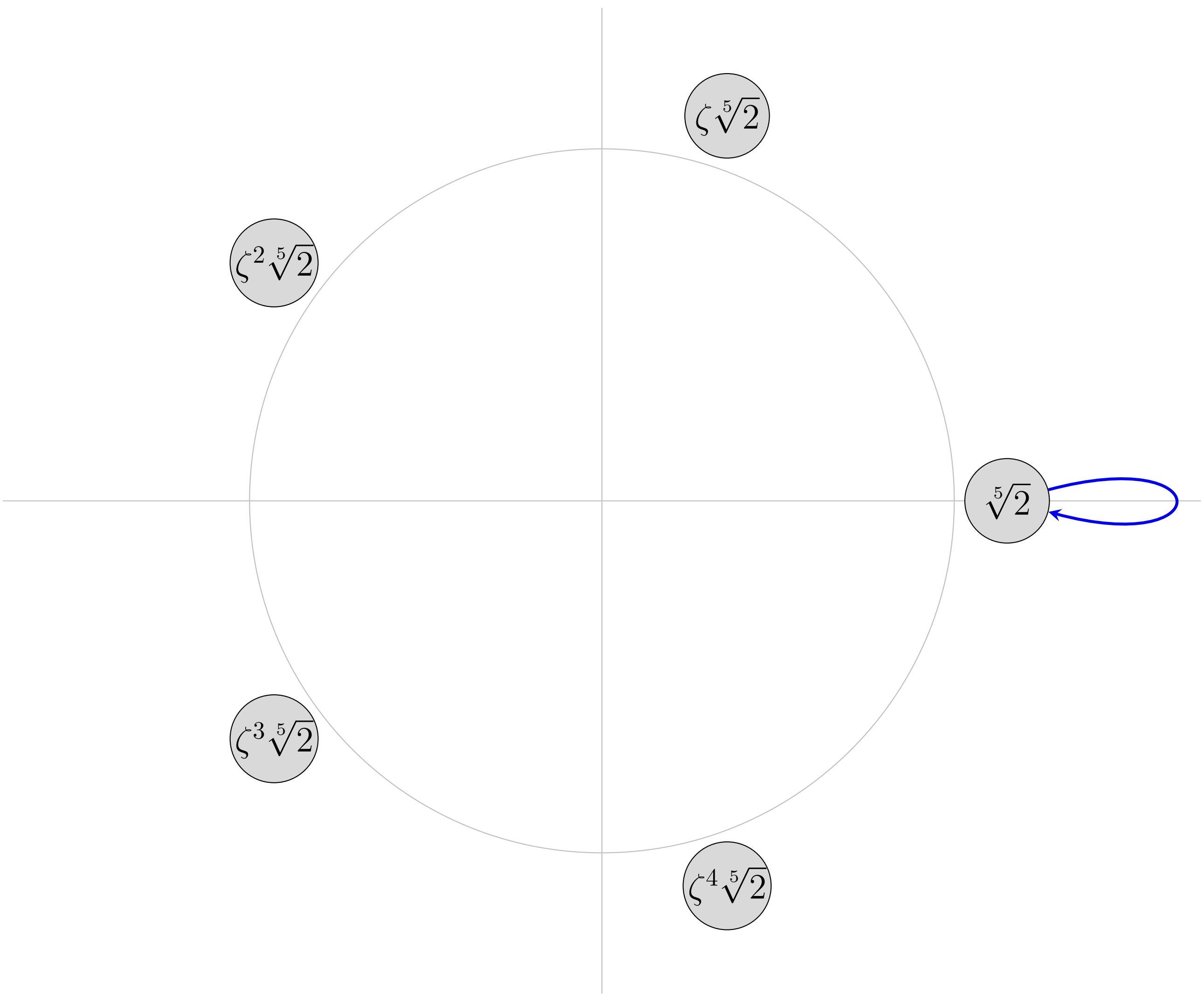
**#2(a)**: The action graph of the Galois group

$$\mathrm{Gal}(x^5-2)=\langle \rho,\sigma \mid \rho^5=1,\ \sigma^4=1,\ \rho\sigma=\sigma\rho^3\rangle \cong \mathrm{AGL}_2(\mathbb{Z}_5),$$

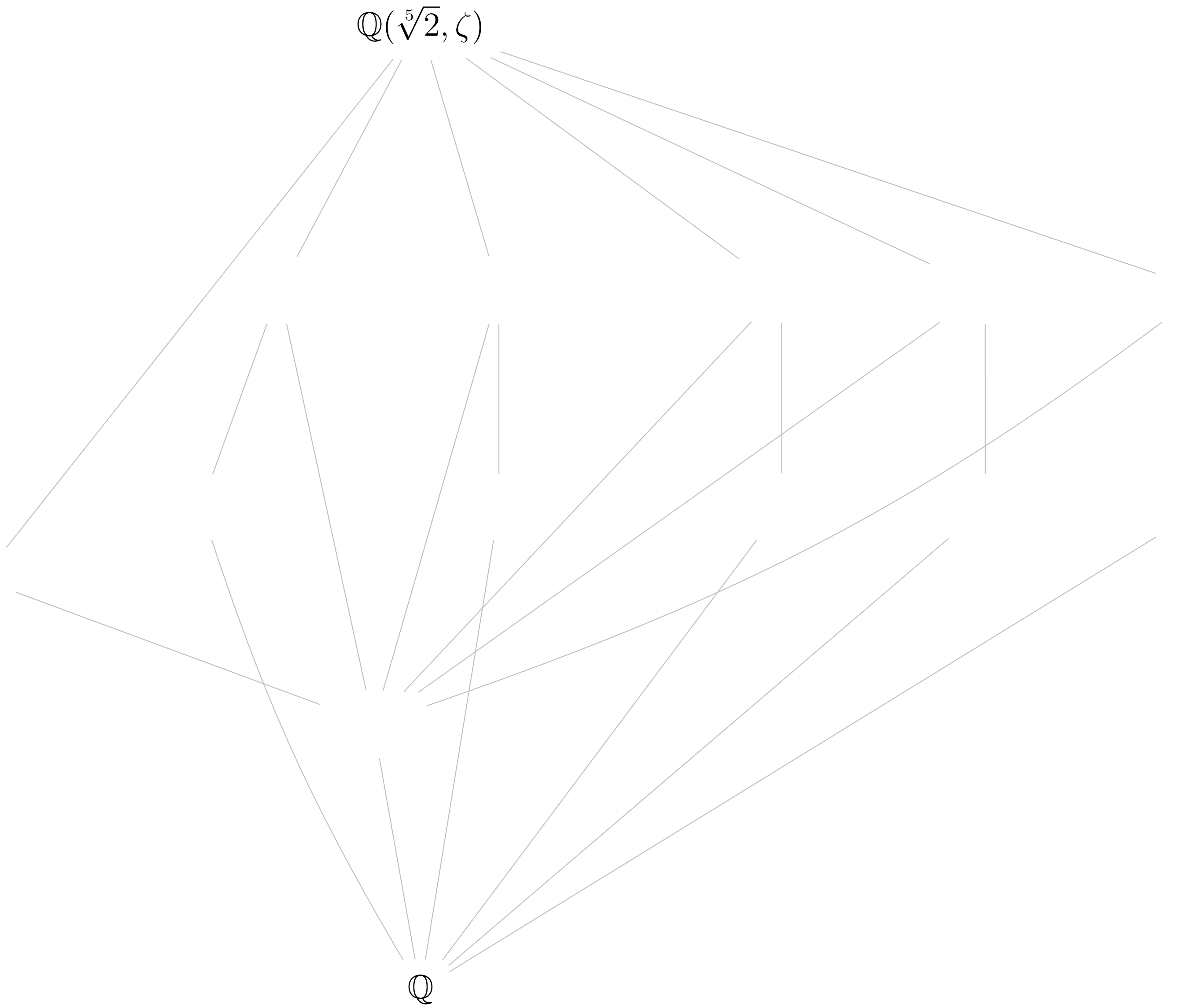
acting on the set $S$ of roots of the polynomial $f(x)=x^5-2$. This is the automorphism group of its splitting field $\mathbb{Q}(\sqrt[5]{2},\zeta)$, where

$$\begin{cases}\rho\colon \sqrt[5]{2}\longmapsto \zeta\sqrt[5]{2}\\ \rho\colon \quad \zeta\longmapsto\zeta\end{cases}\qquad\qquad \begin{cases}\sigma\colon \sqrt[5]{2}\longmapsto \sqrt[5]{2}\\ \sigma\colon \quad \zeta\longmapsto\zeta^2,\end{cases}$$

where $\zeta=e^{2\pi i/5}$, a primitive $5^{\text{th}}$ root of unity.



**#2(b)**: The subfield lattice of $\mathbb{Q}(\sqrt[5]{2}, \zeta)$, for $\zeta = e^{2\pi i/5}$, where each field fixes the corresponding subgroup of $\mathrm{Gal}(x^5 - 2)$ in its subgroup lattice.



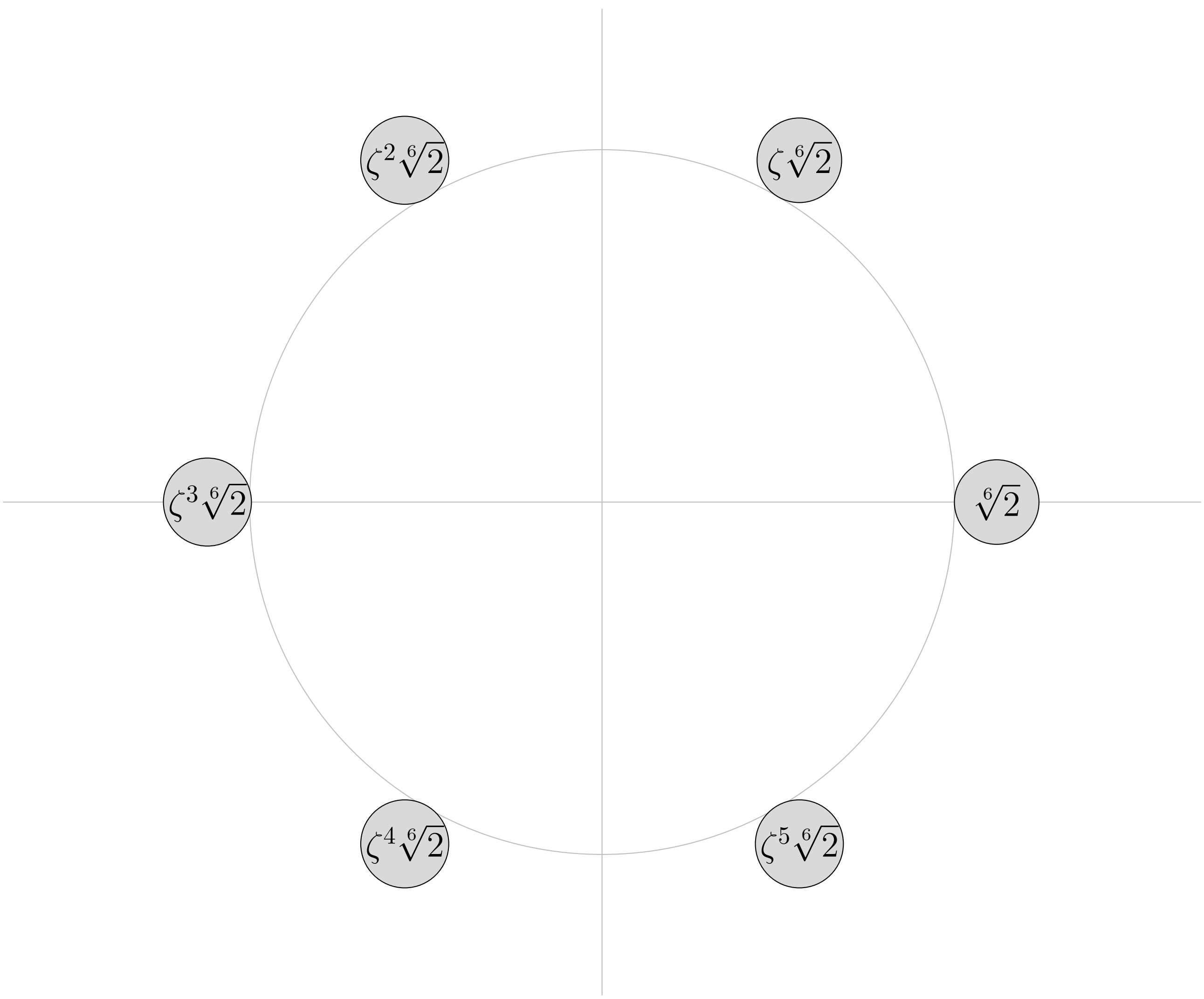
**#3(a)**: The action graph of the Galois group

$$\mathrm{Gal}(x^6-2) = \langle \rho, \sigma \mid \rho^6 = 1,\ \sigma^2 = 1,\ \rho\sigma = \sigma\rho^5 \rangle \cong D_6,$$

acting on the set $S$ of roots of the polynomial $f(x) = x^6 - 2$. This is the automorphism group of its splitting field $\mathbb{Q}(\sqrt[6]{2}, \zeta)$, where

$$\begin{cases} \rho\colon \sqrt[6]{2} \longmapsto \zeta\sqrt[6]{2} \\ \rho\colon \quad \zeta \longmapsto \zeta \end{cases} \qquad \begin{cases} \sigma\colon \sqrt[6]{2} \longmapsto \sqrt[6]{2} \\ \sigma\colon \quad \zeta \longmapsto \zeta^5 = \overline{\zeta}, \end{cases}$$

where $\zeta = e^{2\pi i/6}$, a primitive $6^{\text{th}}$ root of unity.



**#3(b)**: The subfield lattice of $\mathbb{Q}(\sqrt[6]{2}, \zeta)$, for $\zeta = e^{2\pi i/6}$, where each field fixes the corresponding subgroup of $\mathrm{Gal}(x^6 - 2)$ in its subgroup lattice.