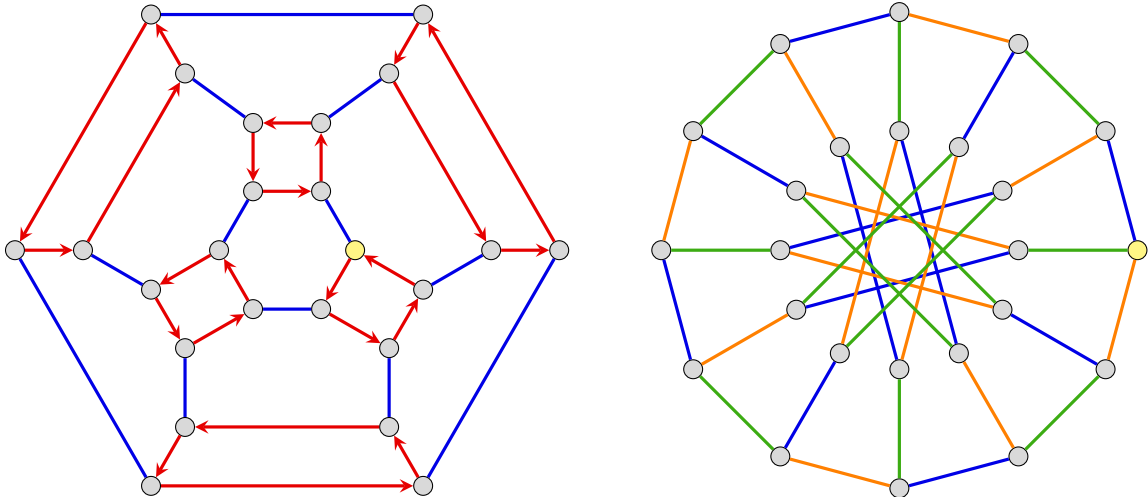


1. Below are two Cayley diagrams for the symmetric group

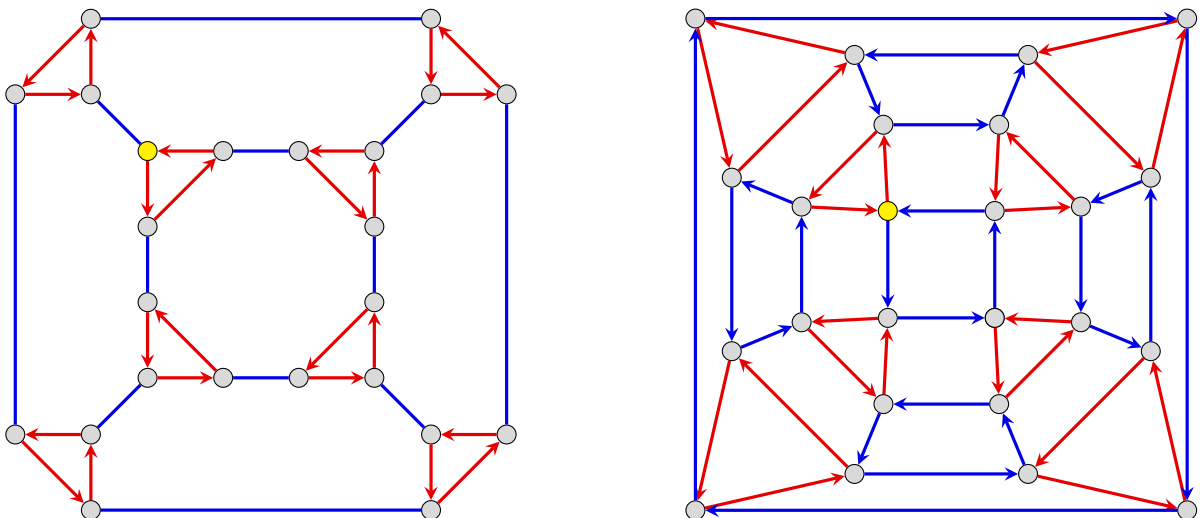
$$S_4 = \langle (1234), (12) \rangle = \langle (12), (13), (14) \rangle.$$

At left is a truncated octahedron, called the *permutohedron*. At right is the *Nauru graph*.



Carry out the following steps, taking the yellow node to represent the identity.

- On both diagrams, label the nodes by elements of  $S_4$ , written in cycle notation as a product of disjoint cycles.
  - On the Nauru graph, label the nodes with permutations of the word **1234**, where  $(i j)$  swaps the  $i^{\text{th}}$  and  $j^{\text{th}}$  coordinates.
  - On a separate copy of the Nauru graph, label the nodes with permutations of **1234**, where  $(i j)$  swaps the *numbers*  $i$  and  $j$ .
2. Two Cayley diagrams for the symmetric group  $S_4$  arranged on flattened Archimedean solids – the truncated cube (left) and the rhombicuboctahedron (right).

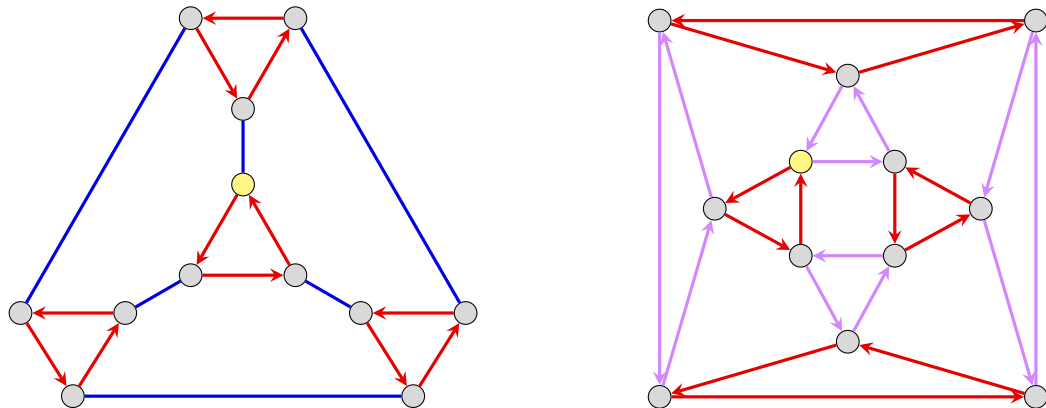


Determine what generating sets will yield these Cayley diagrams. Then, label the nodes with permutations in cycle notation, written as a product of disjoint cycles.

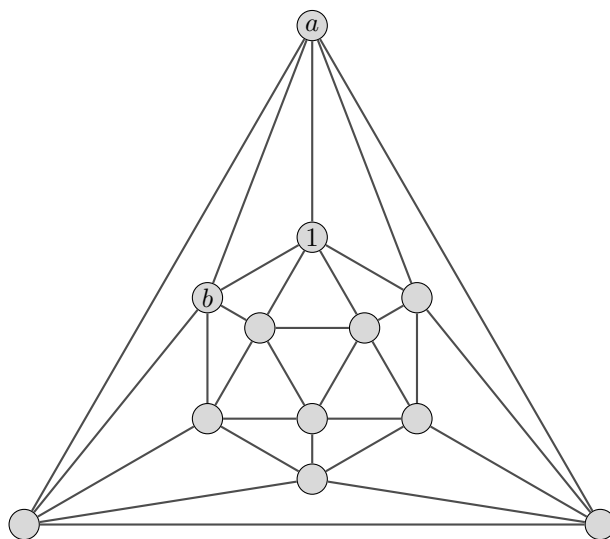
3. The *alternating* group  $A_4$  is the subgroup of  $S_4$  that consists of the even permutations. Two Cayley diagrams are shown below, for presentations

$$A_4 = \langle (123), (12)(34) \rangle = \langle (123), (234) \rangle.$$

Label the nodes of these diagrams with elements of  $A_4$  in cycle notation, written as a product of disjoint cycles.



4. Draw the Cayley diagram of the group  $G = \langle a, b, c \mid a^2 = b^3 = c^3 = abc = 1 \rangle$  on the skeleton of the icosahedron, shown below, and label the nodes with elements written using  $a, b,$  and  $c$ .



There are five groups of order 12: the abelian groups  $C_{12}$  and  $C_6 \times C_2$ , the dihedral group  $D_6$ , the alternating group  $A_4$ , and the dicyclic group  $\text{Dic}_6$ . Determine which group  $G$  is isomorphic to, and then re-draw this Cayley diagram with the nodes labeled with elements of that group.

5. Prove that if  $g^2 = e$  for all  $g \in G$ , then  $G$  must be abelian.