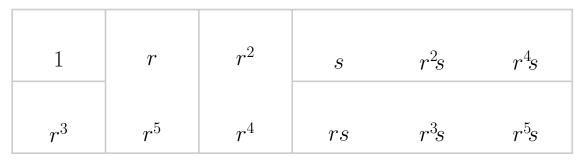
## Supplemental material: Visual Algebra (Math 4120), HW 11

#1(a): Action graph and fixed point table of  $Dic_6 = \langle r, s \rangle$  acting on itself by conjugation.



	1											
	1	r	$r^2$	$r^3$	$r^4$	$r^5$	s	rs	$r^2s$	$r^3s$	$r^4s$	$r^5s$
1												
r												
$r^2$												
$r^3$												
$r^4$												
$r^5$												
s												
rs												
$r^2s$												
$r^3s$												
$r^4s$												
$r^5s$												

#1(b): Action graph and fixed point table of the inner automorphism group  $G = \operatorname{Inn}(\operatorname{Dic}_6) = \langle \varphi_r, \varphi_s \rangle \cong \operatorname{Dic}_6 / Z(\operatorname{Dic}_6) \cong \operatorname{Dic}_6 / \langle r^3 \rangle \cong D_3$  acting on  $S = \operatorname{Dic}_6$  by conjugation.

1	r	$r^2$	s	$r^2\!s$	$r^4\!s$
$r^3$	$r^5$	$r^4$	rs	$r^3\!s$	$r^5\!s$

$r^2$	$r^5$	$r^2s$	$r^5s$
r	$r^4$	rs	$r^4s$
1	$r^3$	s	$r^3s$

Action graph

 $\mathrm{Dic}_6/\langle r^3
angle$ 

	1	r	$r^2$	$r^3$	$r^4$	$r^5$	S	rs	$r^2s$	$r^3\!s$	$r^4\!s$	$r^5\!s$
Id												
$arphi_r$												
$arphi_{r^2}$												
$arphi_s$												
$arphi_{rs}$												
$arphi_{r^2\!s}$												

 $\#1(\mathbf{c})$ : Partition of  $\mathrm{Aut}(\mathrm{Dic}_6) = \langle \varphi_r, \varphi_s, \omega \rangle$  into cosets of  $\mathrm{Inn}(\mathrm{Dic}_6)$ .

 $Inn(Dic_6) = \langle \varphi_r, \varphi_s \rangle$  $\operatorname{Inn}(\operatorname{Dic}_6)\omega$  $r^2$ 0 1  $\binom{1}{1}$  $\bigcap_{r^2s}$  $r^2$  $r^4s$ r $r^2s$  $r^4s$ Id  $\omega$  $\bigcap_{r^3}$  $\bigcap_{r^3s}$  $\bigcap_{r^5s}$  $\bigcap_{r^3}$  $\mathop{\bigcirc}_{rs}$  $r^5$  $r^5s$  $\bigcap_1$  $r^2$  $r^2$  $r^2s$ r $r^4s$ r $r^2s$ s $r^4s$  $\varphi_r \omega$  $r^3$  $r^3s$  $r^5$  $r^5$  $r^5s$  $r^5s$ rs $\bigcap_1$  $r^2$  $r^2$ r $r^2s$  $r^4s$ r $r^2s$ s $\varphi_{r^2}$  $\varphi_{r^2}\omega$  $r^3$  $r^3s$  $r^5s$  $r^5$  $r^5s$ rs $\binom{1}{1}$  $r^2$  $r^2$  $r^2s$  $r^4s$ rr $r^2s$ s $\varphi_s$  $\varphi_f \omega$  $r^3$  $r^3$  $r^3s$  $r^5s$  $r^5$  $r^5s$ rs $\bigcap_1$  $r^2$  $r^2$ r $r^2s$  $r^4s$ rs $r^2s$  $\varphi_{rs}$  $\varphi_{rs}\omega$  $r^3$  $r^3s$  $r^5s$  $r^5$  $r^5s$ rs $\bigcap_1$  $r^2$  $r^2$ r $r^2s$ r

 $r^4s$ 

 $r^5s$ 

 $\bigcap_{r^3}$ 

 $r^5$ 

 $r^4$ 

s

rs

 $r^4$ 

 $r^3s$ 

 $arphi_{r^{2_{\!s}}}$ 

 $r^2s$ 

 $r^3s$ 

 $r^5s$ 

 $arphi_{r^2\!s}\omega$ 

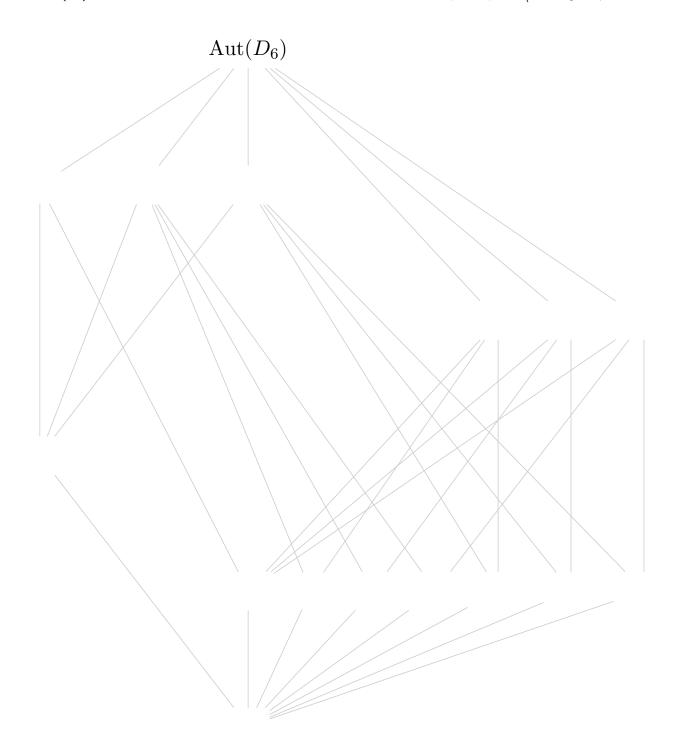
#1(c): Action graph and fixed point table of  $G = \operatorname{Aut}(\operatorname{Dic}_6) = \langle \varphi_r, \varphi_s, \omega \rangle$  acting on  $S = \operatorname{Dic}_6$ , where  $\omega$  is the outer automorphism defined by

$$\omega \colon \operatorname{Dic}_6 \longrightarrow \operatorname{Dic}_6, \qquad \omega(r) = r, \quad \omega(s) = s^{-1} = r^3 s.$$

1	r	$r^2$	s	$r^2\!s$	$r^4\!s$
$r^3$	$r^5$	$r^4$	rs	$r^3\!s$	$r^5\!s$

	1	r	$r^2$	$r^3$	$r^4$	$r^5$	s	rs	$r^2s$	$r^3s$	$r^4s$	$r^5s$
Id												
$arphi_r$												
$arphi_{r^2}$												
$arphi_s$												
$arphi_{rs}$												
$arphi_{r^2\!s}$												
$\omega$												
$\varphi_r \omega$												
$arphi_{r^2}\omega$												
$arphi_s\omega$												
$\varphi_{rs}\omega$												
$arphi_{r^2\!s}\omega$												

 $\#1(\mathbf{d})$ : Cayley graph and subgroup lattice of  $\mathrm{Aut}(\mathrm{Dic}_6) = \langle \varphi_r, \varphi_f, \omega \rangle \cong D_6$ .



#1(e): Action graph and fixed point table of the action of Aut(Dic<sub>6</sub>) =  $\langle \varphi_r, \varphi_s, \omega \rangle$  on the conjugacy classes of Dic<sub>6</sub>.





(cl(r))

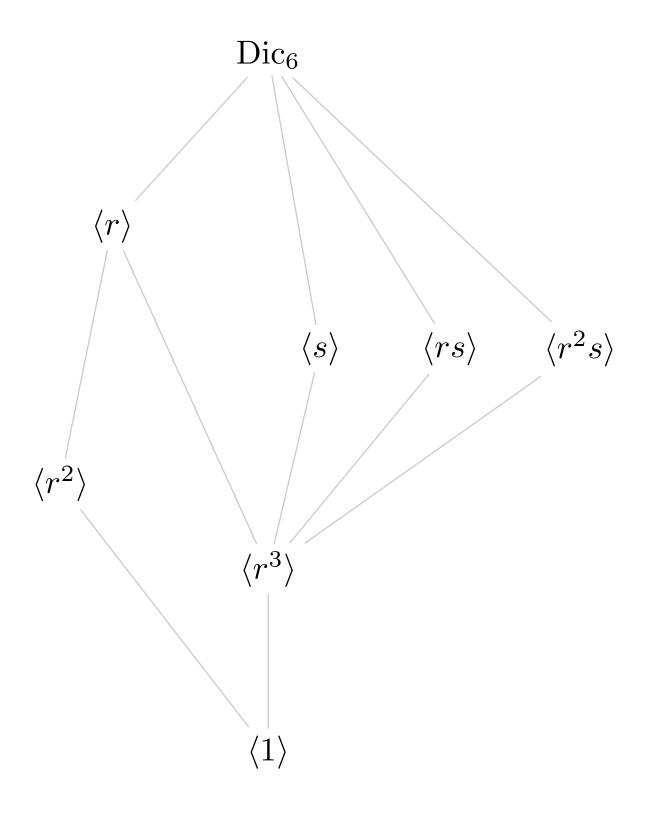


 $(\operatorname{cl}(s))$ 



	$\operatorname{cl}(1)$	$\operatorname{cl}(r^3)$	$\operatorname{cl}(r)$	$\operatorname{cl}(r^2)$	$\operatorname{cl}(s)$	$\operatorname{cl}(rs)$
Id						
$arphi_r$						
$arphi_{r^2}$						
$arphi_s$						
$arphi_{rs}$						
$arphi_{r^{2_{\!s}}}$						
$\omega$						
$arphi_r \omega$						
$arphi_{r^2}\omega$						
$arphi_s\omega$						
$arphi_{rs}\omega$						
$arphi_{r^{2_{\!s}}}\!\omega$						

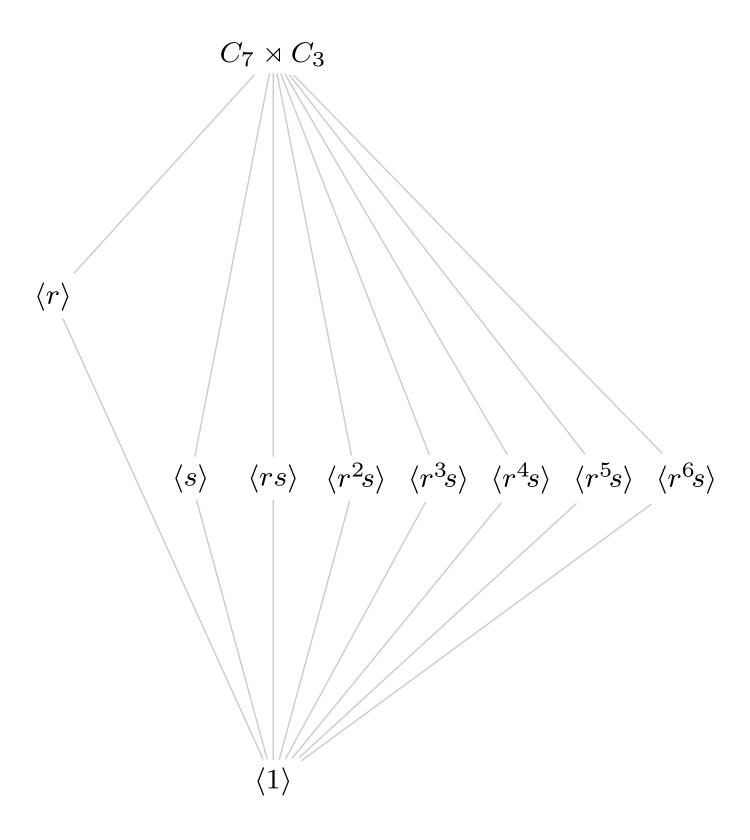
#1(f): Action graph of Dic<sub>6</sub> acting on its subgroups by conjugation.



#1(f): Fixed point table of  $\operatorname{Dic}_6$  acting on its subgroups by conjugation.

	$\operatorname{Dic}_6$	$\langle r \rangle$	$\langle s \rangle$	$\langle rs \rangle$	$\langle r^2 s \rangle$	$\langle r^2 \rangle$	$\langle r^3 \rangle$	$\langle 1 \rangle$
1								
r								
$r^2$								
$r^3$								
$r^4$								
$r^5$								
s								
rs								
$r^2s$								
$r^3s$								
$r^3s$ $r^4s$ $r^5s$								
$r^5s$								

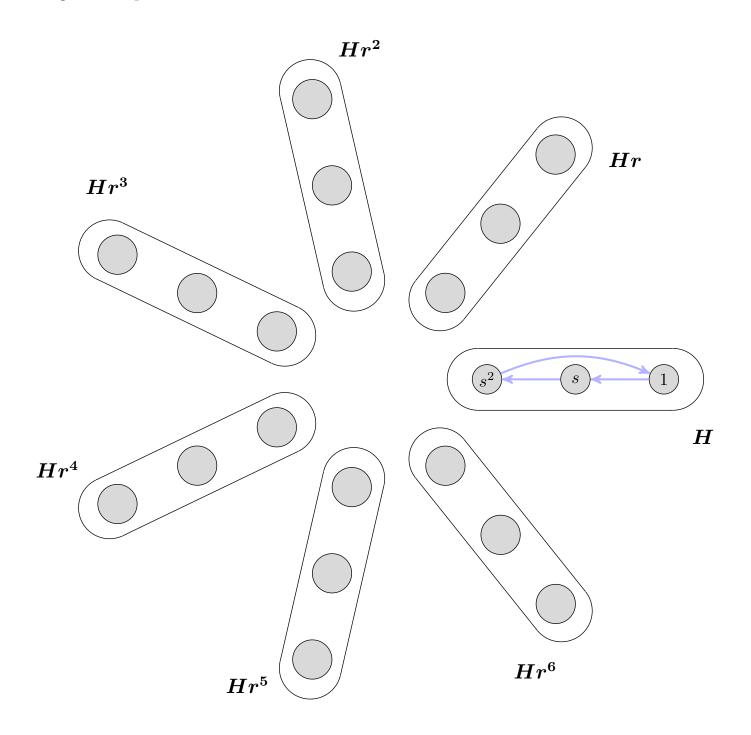
 $\#2(\mathbf{a})$ : Action graph of  $C_7 \rtimes C_3$  acting on its subgroups by conjugation.



#2(a): Fixed point table of  $C_7 \rtimes C_3$  acting on its subgroups by conjugation.

	$C_7 \rtimes C_3$	$\langle r \rangle$	$\langle s \rangle$	$\langle rs \rangle$	$\langle r^2 s \rangle$	$\langle r^3 s \rangle$	$\langle r^4 s \rangle$	$\langle r^5 s \rangle$	$\langle r^6 s \rangle$	$\langle 1 \rangle$
1										
r										
$r^2$										
$r^3$										
$r^4$										
$r^5$										
$r^6$										
s										
rs										
$r^2s$										
$r^3s$										
$r^4s$										
$r^5s$										
$r^6s$										
$s^2$										
$rs^2$										
$r^2s^2$										
$r^3s^2$										
$r^4s^2$										
$r^5s^2$										
$r^6s^2$										

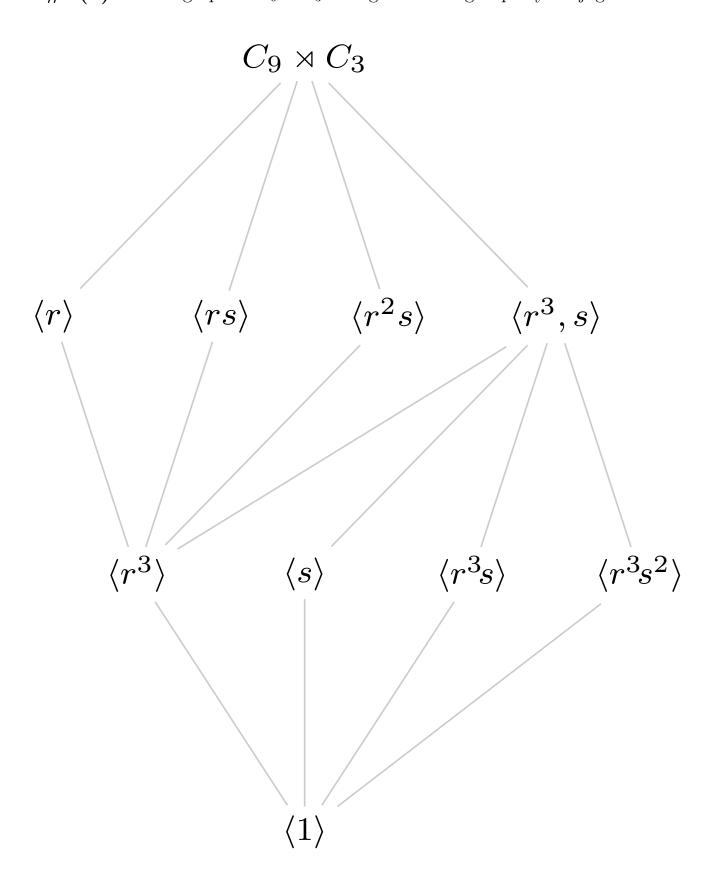
 $\#2(\mathbf{b})$ : Action graph of  $C_7 \rtimes C_3 = \langle r, s \rangle$  acting on the right cosets of  $H = \langle s \rangle$  by right multiplication.



 $\#2(\mathbf{b})$ : Fixed point table of  $C_7 \rtimes C_3 = \langle r, s \rangle$  acting on the right cosets of  $H = \langle s \rangle$  by right multiplication.

	H	Hr	$Hr^2$	$Hr^3$	$Hr^4$	$Hr^5$	$Hr^6$
1							
r							
$r^2$							
$r^3$							
$r^4$							
$r^5$							
$r^6$							
s							
rs							
$r^2s$							
$r^3s$							
$r^4s$							
$r^5s$							
$r^6s$							
$s^2$							
$rs^2$							
$r^2s^2$							
$r^3s^2$							
$r^4s^2$							
$r^5s^2$							
$r^6s^2$							

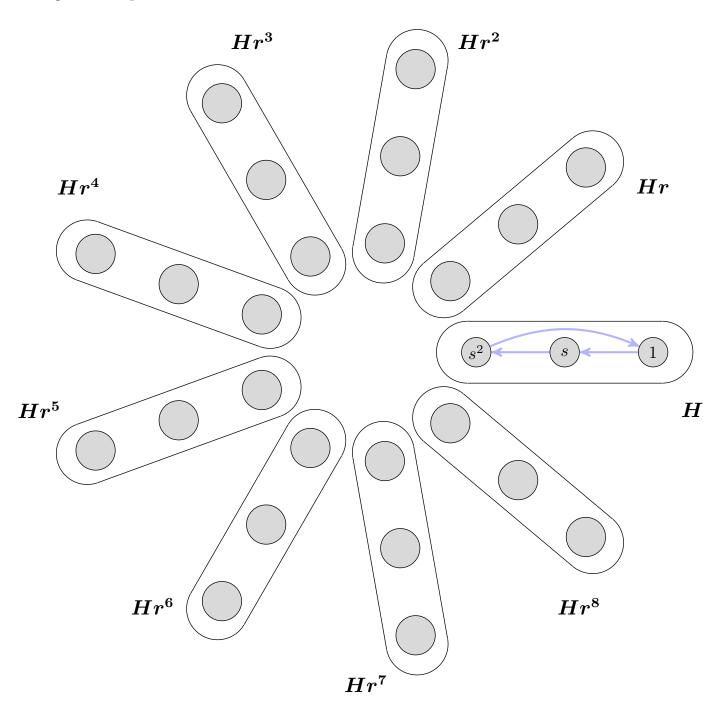
 $\#2(\mathbf{a})$ : Action graph of  $C_9 \rtimes C_3$  acting on its subgroups by conjugation.



#2(a): Fixed point table of  $C_9 \rtimes C_3$  acting on its subgroups by conjugation.

	$C_9 \rtimes C_3$	$\langle r \rangle$	$\langle rs \rangle$	$\langle r^2 s \rangle$	$\langle r^3, s \rangle$	$\langle r^3 \rangle$	$\langle s \rangle$	$\langle r^3 s \rangle$	$\langle r^3 s^2 \rangle$	$\langle 1 \rangle$
1										
r										
$r^2$										
$r^3$										
$r^4$										
$r^5$										
$r^6$										
$r^7$										
$r^8$										
s										
rs										
$r^2s$										
$r^3s$										
$r^4s$										
$r^5s$										
$r^6s$										
$r^7s$										
$r^8s$										
$s^2$										
$rs^2$										
$r^2s^2$										
$r^3s^2$										
$r^4s^2$										
$r^5s^2$										
$r^6s^2$										
$r^7s^2$										
$r^8s^2$										

 $\#2(\mathbf{b})$ : Action graph of  $C_9 \rtimes C_3 = \langle r, s \rangle$  acting on the right cosets of  $H = \langle s \rangle$  by right multiplication.



 $\#2(\mathbf{b})$ : Fixed point table of  $C_7 \rtimes C_3 = \langle r, s \rangle$  acting on the right cosets of  $H = \langle s \rangle$  by right multiplication.

	H	Hr	$Hr^2$	$Hr^3$	$Hr^4$	$Hr^5$	$Hr^6$	$Hr^7$	$Hr^8$
1									
r									
$r^2$									
$r^3$									
$r^4$									
$r^5$									
$r^6$									
$r^7$									
$r^8$									
s									
rs									
$r^2s$									
$r^3s$									
$r^4s$									
$r^5s$									
$r^6s$									
$r^7s$									
$r^8s$									
$s^2$									
$rs^2$									
$r^2s^2$									
$r^3s^2$									
$r^4s^2$									
$r^5s^2$									
$r^6s^2$									
$r^7s^2$									
$r^8s^2$									