Supplemental material: Visual Algebra (Math 4120), HW 11

#1(a): Action graph and fixed point table of $\mathrm{Dic}_6 = \langle r, s \rangle$ acting on itself by conjugation.

1	r	r^2	s	r^2s	r^4s
r^3	r^5	r^4	rs	r^3s	r^5s

	1	r	r^2	r^3	r^4	r^5	s	rs	r^2s	r^3s	r^4s	r^5s
1												
r												
r^2												
r^3												
r^4												
r^5												
s												
rs												
r^2s												
r^3s												
r^4s												
r^5s												

#1(b): Action graph and fixed point table of the inner automorphism group

$$G = \mathrm{Inn}(\mathrm{Dic}_6) = \langle \varphi_r, \varphi_s \rangle \cong \mathrm{Dic}_6 / Z(\mathrm{Dic}_6) \cong \mathrm{Dic}_6 / \langle r^3 \rangle \cong D_3$$

acting on $S = \mathrm{Dic}_6$ by conjugation.

1	r	r^2	s	r^2s	r^4s
r^3	r^5	r^4	rs	r^3s	r^5s

Action graph

r^2	r^5	r^2s	r^5s
r	r^4	rs	r^4s
1	r^3	s	r^3s

$\mathrm{Dic}_6/\langle r^3 \rangle$

	1	r	r^2	r^3	r^4	r^5	s	rs	r^2s	r^3s	r^4s	r^5s
Id												
φ_r												
φ_{r^2}												
φ_s												
φ_{rs}												
φ_{r^2s}												

#1(c): Partition of $\mathrm{Aut}(\mathrm{Dic}_6) = \langle \varphi_r, \varphi_s, \omega \rangle$ into cosets of $\mathrm{Inn}(\mathrm{Dic}_6)$.

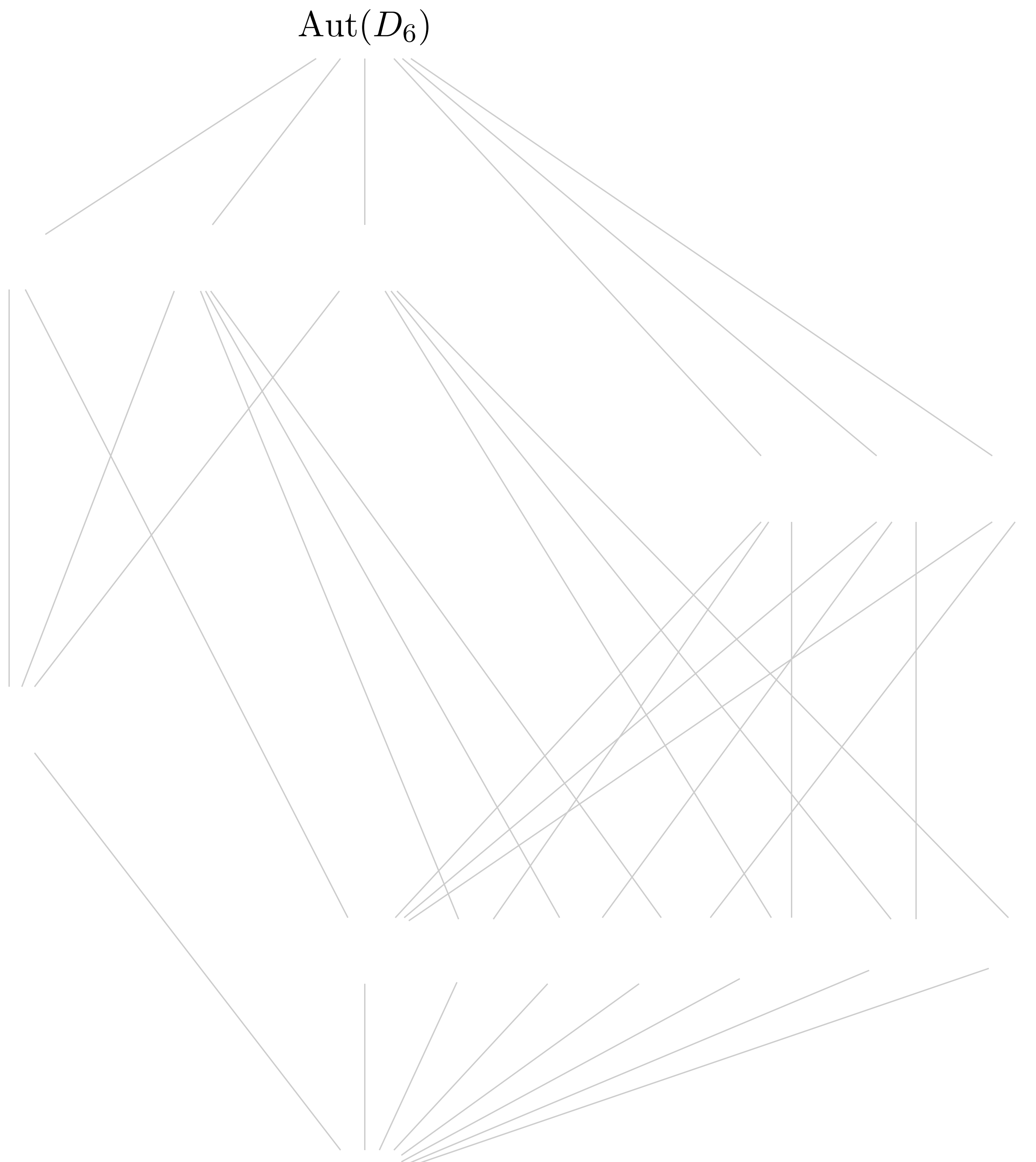
#1(c): Action graph and fixed point table of $G = \mathrm{Aut}(\mathrm{Dic}_6) = \langle \varphi_r, \varphi_s, \omega \rangle$ acting on $S = \mathrm{Dic}_6$, where ω is the outer automorphism defined by

$$\omega \colon \mathrm{Dic}_6 \longrightarrow \mathrm{Dic}_6, \qquad \omega(r) = r, \quad \omega(s) = s^{-1} = r^3 s.$$

1	r	r^2	s	$r^2 s$	$r^4 s$
r^3	r^5	r^4	rs	$r^3 s$	$r^5 s$

	1	r	r^2	r^3	r^4	r^5	s	rs	$r^2 s$	$r^3 s$	$r^4 s$	$r^5 s$
Id												
φ_r												
φ_{r^2}												
φ_s												
φ_{rs}												
$\varphi_{r^2 s}$												
ω												
$\varphi_r \omega$												
$\varphi_{r^2} \omega$												
$\varphi_s \omega$												
$\varphi_{rs} \omega$												
$\varphi_{r^2 s} \omega$												

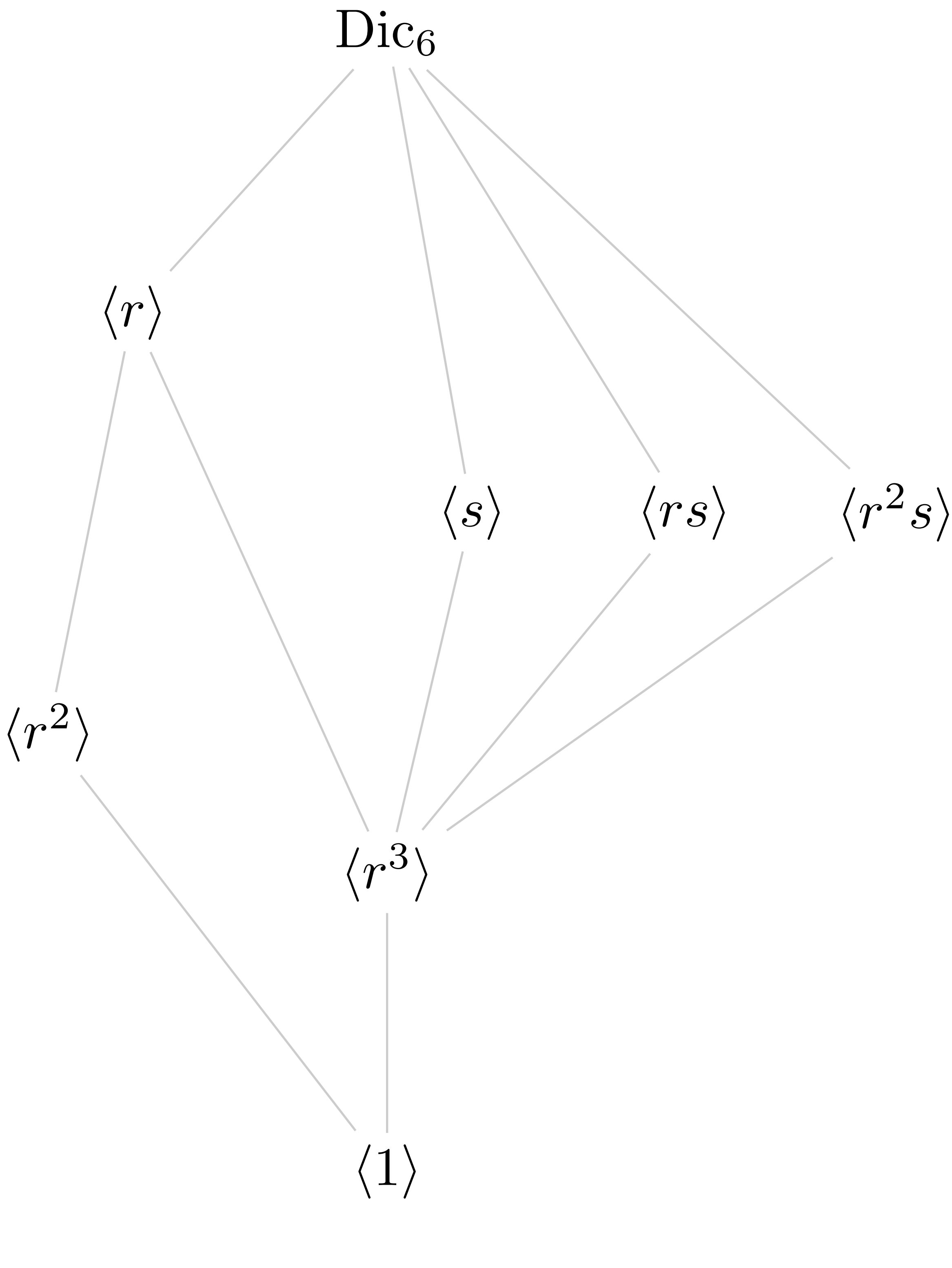
#1(d): Cayley graph and subgroup lattice of $\mathrm{Aut}(\mathrm{Dic}_6) = \langle \varphi_r, \varphi_f, \omega \rangle \cong D_6$.



#1(e): Action graph and fixed point table of the action of $\mathrm{Aut}(\mathrm{Dic}_6) = \langle \varphi_r, \varphi_s, \omega \rangle$ on the conjugacy classes of Dic_6.

	$\mathrm{cl}(1)$	$\mathrm{cl}(r^3)$	$\mathrm{cl}(r)$	$\mathrm{cl}(r^2)$	$\mathrm{cl}(s)$	$\mathrm{cl}(rs)$
Id						
φ_r						
φ_{r^2}						
φ_s						
φ_{rs}						
φ_{r^2s}						
ω						
$\varphi_r\omega$						
$\varphi_{r^2}\omega$						
$\varphi_s\omega$						
$\varphi_{rs}\omega$						
$\varphi_{r^2s}\omega$						

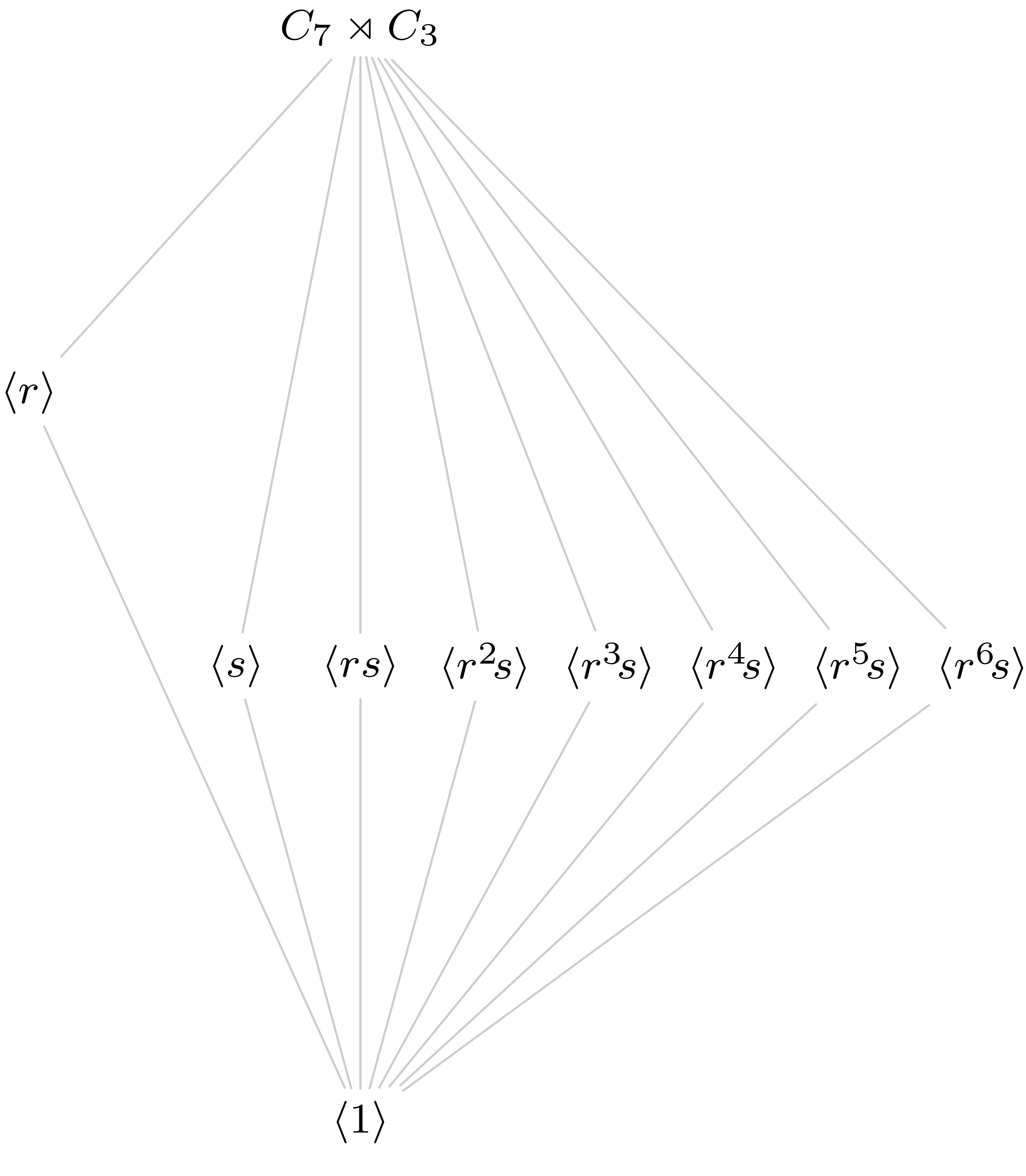
#1(f): Action graph of Dic_6 acting on its subgroups by conjugation.



#1(f): Fixed point table of Dic_6 acting on its subgroups by conjugation.

	Dic_6	$\langle r \rangle$	$\langle s \rangle$	$\langle rs \rangle$	$\langle r^2 s \rangle$	$\langle r^2 \rangle$	$\langle r^3 \rangle$	$\langle 1 \rangle$
1								
r								
r^2								
r^3								
r^4								
r^5								
s								
rs								
$r^2 s$								
$r^3 s$								
$r^4 s$								
$r^5 s$								

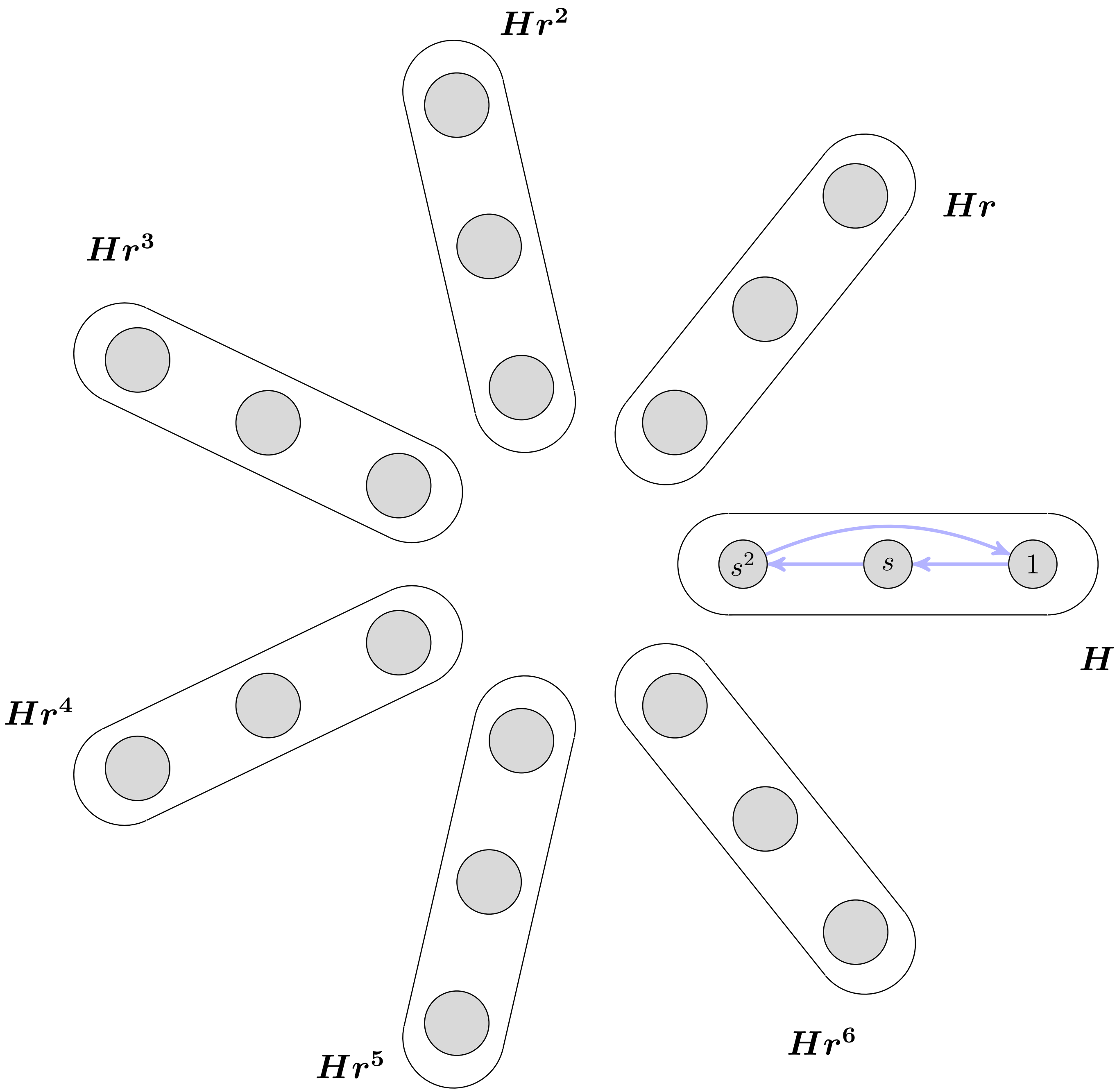
#2(a): Action graph of $C_7 \rtimes C_3$ acting on its subgroups by conjugation.



#2(a): Fixed point table of $C_7 \rtimes C_3$ acting on its subgroups by conjugation.

	$C_7 \rtimes C_3$	$\langle r \rangle$	$\langle s \rangle$	$\langle rs \rangle$	$\langle r^2 s \rangle$	$\langle r^3 s \rangle$	$\langle r^4 s \rangle$	$\langle r^5 s \rangle$	$\langle r^6 s \rangle$	$\langle 1 \rangle$
1										
r										
r^2										
r^3										
r^4										
r^5										
r^6										
s										
rs										
$r^2 s$										
$r^3 s$										
$r^4 s$										
$r^5 s$										
$r^6 s$										
s^2										
rs^2										
$r^2 s^2$										
$r^3 s^2$										
$r^4 s^2$										
$r^5 s^2$										
$r^6 s^2$										

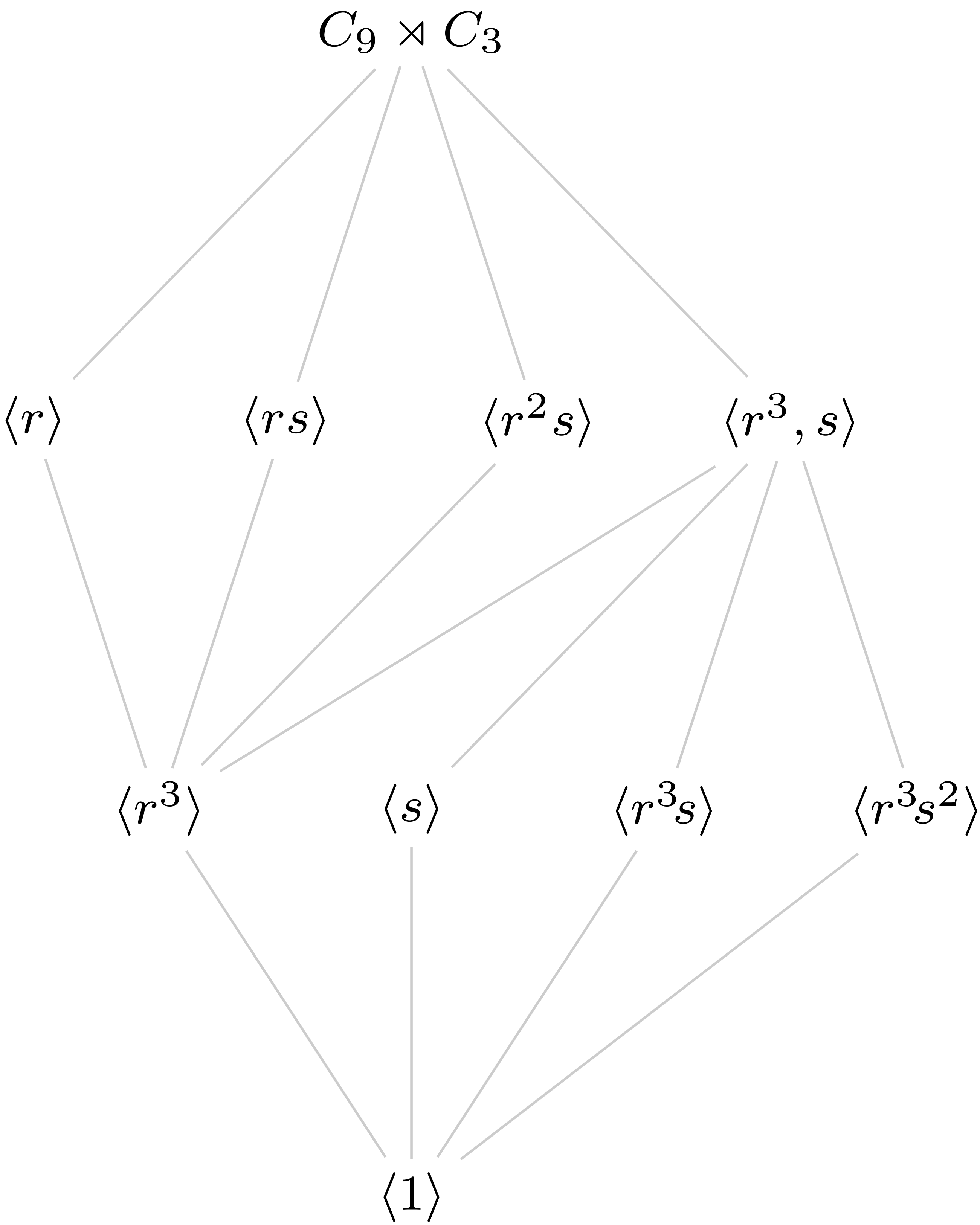
#2(b): Action graph of $C_7 \rtimes C_3 = \langle r, s \rangle$ acting on the right cosets of $H = \langle s \rangle$ by right multiplication.



#2(b): Fixed point table of $C_7 \rtimes C_3 = \langle r, s \rangle$ acting on the right cosets of $H = \langle s \rangle$ by right multiplication.

	H	Hr	Hr^2	Hr^3	Hr^4	Hr^5	Hr^6
1							
r							
r^2							
r^3							
r^4							
r^5							
r^6							
s							
rs							
r^2s							
r^3s							
r^4s							
r^5s							
r^6s							
s^2							
rs^2							
r^2s^2							
r^3s^2							
r^4s^2							
r^5s^2							
r^6s^2							

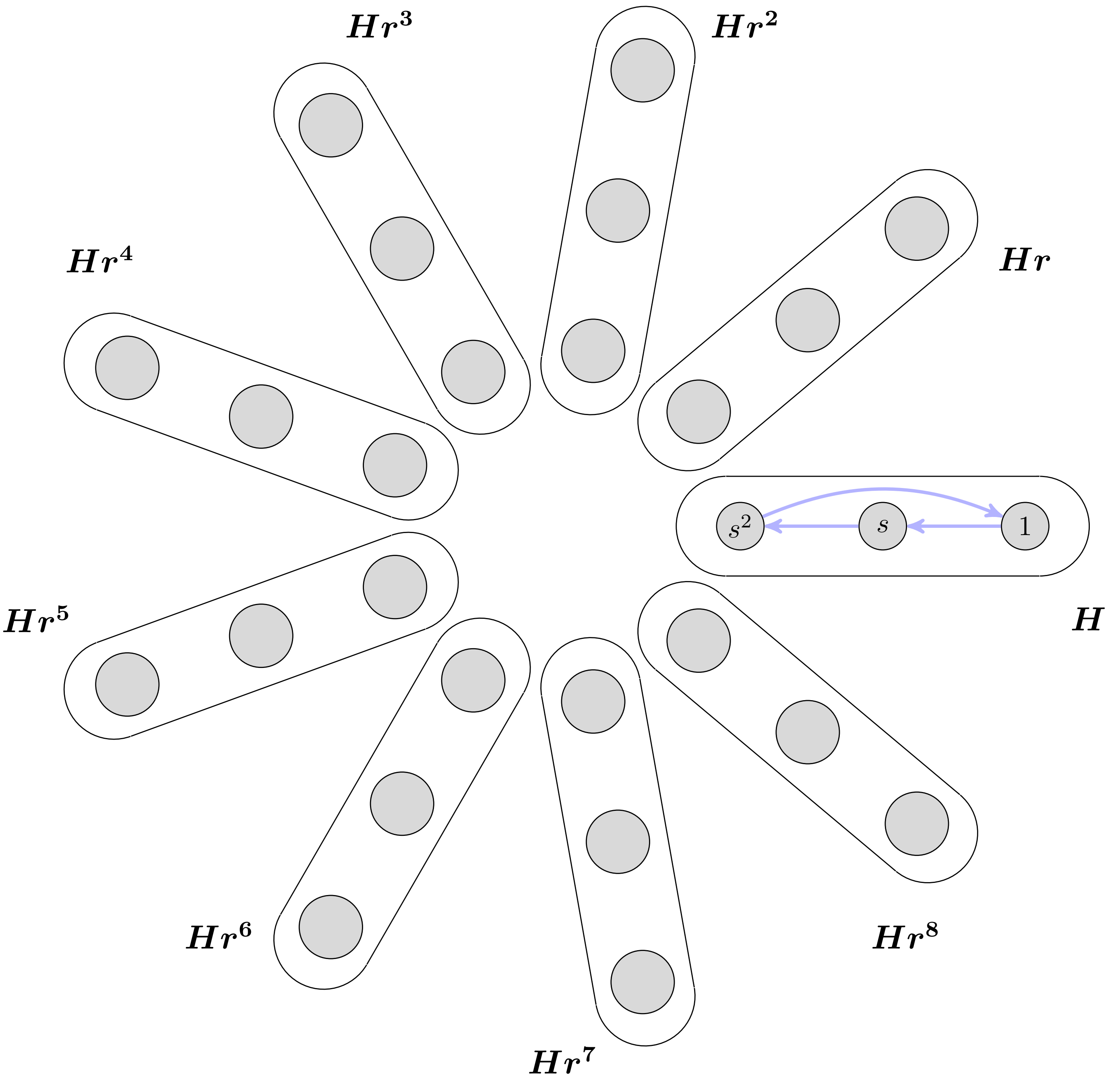
#2(a): Action graph of $C_9 \rtimes C_3$ acting on its subgroups by conjugation.



#2(a): Fixed point table of $C_9 \rtimes C_3$ acting on its subgroups by conjugation.

	$C_9 \rtimes C_3$	$\langle r \rangle$	$\langle rs \rangle$	$\langle r^2 s \rangle$	$\langle r^3, s \rangle$	$\langle r^3 \rangle$	$\langle s \rangle$	$\langle r^3 s \rangle$	$\langle r^3 s^2 \rangle$	$\langle 1 \rangle$
1										
r										
r^2										
r^3										
r^4										
r^5										
r^6										
r^7										
r^8										
s										
rs										
$r^2 s$										
$r^3 s$										
$r^4 s$										
$r^5 s$										
$r^6 s$										
$r^7 s$										
$r^8 s$										
s^2										
rs^2										
$r^2 s^2$										
$r^3 s^2$										
$r^4 s^2$										
$r^5 s^2$										
$r^6 s^2$										
$r^7 s^2$										
$r^8 s^2$										

#2(b): Action graph of $C_9 \rtimes C_3 = \langle r, s \rangle$ acting on the right cosets of $H = \langle s \rangle$ by right multiplication.



#2(b): Fixed point table of $C_7 \rtimes C_3 = \langle r, s \rangle$ acting on the right cosets of $H = \langle s \rangle$ by right multiplication.

	H	Hr	Hr^2	Hr^3	Hr^4	Hr^5	Hr^6	Hr^7	Hr^8
1									
r									
r^2									
r^3									
r^4									
r^5									
r^6									
r^7									
r^8									
s									
rs									
r^2s									
r^3s									
r^4s									
r^5s									
r^6s									
r^7s									
r^8s									
s^2									
rs^2									
r^2s^2									
r^3s^2									
r^4s^2									
r^5s^2									
r^6s^2									
r^7s^2									
r^8s^2									