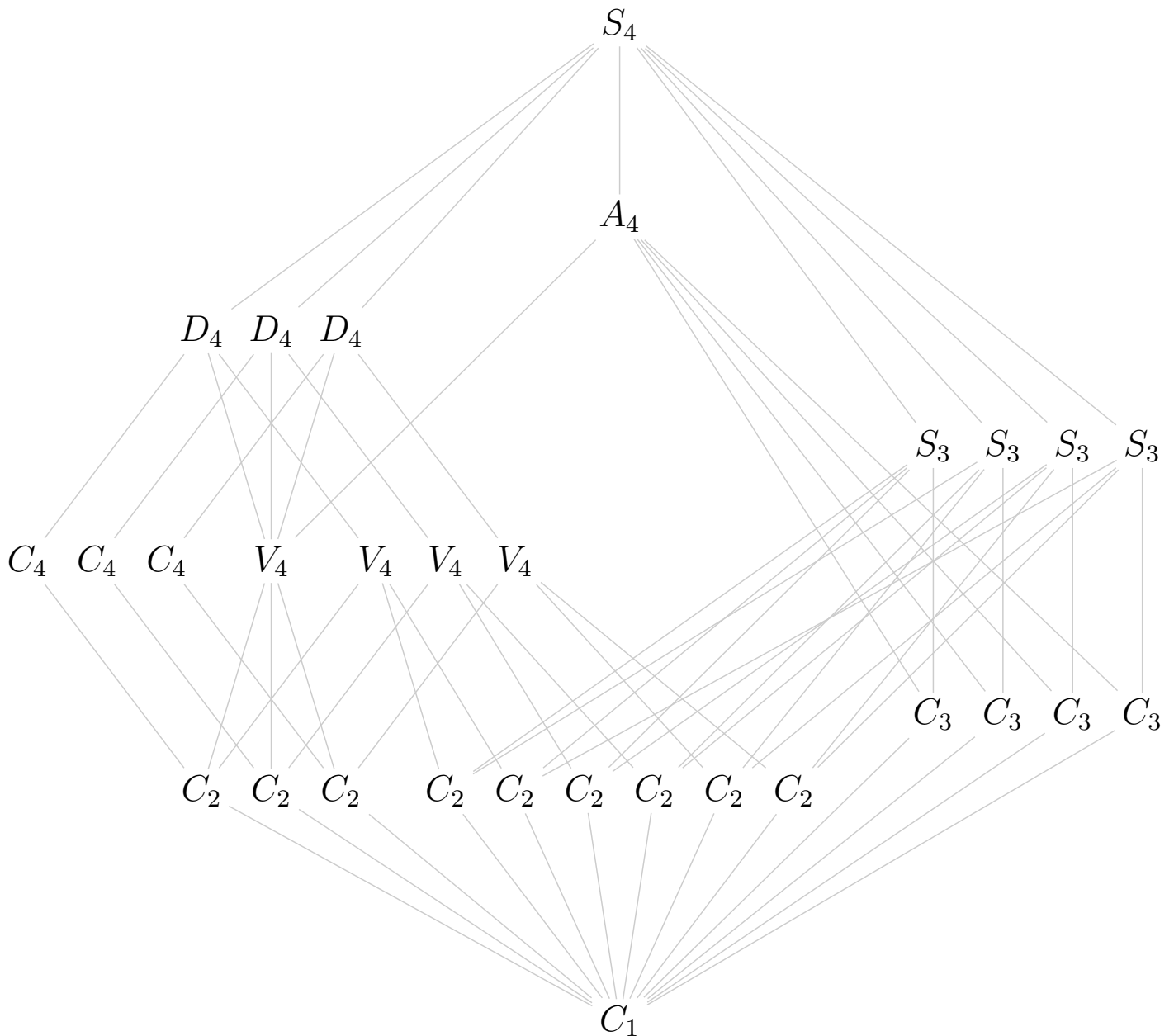


## Supplemental material: Visual Algebra (Math 4120), HW 12

**#1(a):** The subgroup lattice of the symmetric group  $S_4$ , partitioned into conjugacy classes.

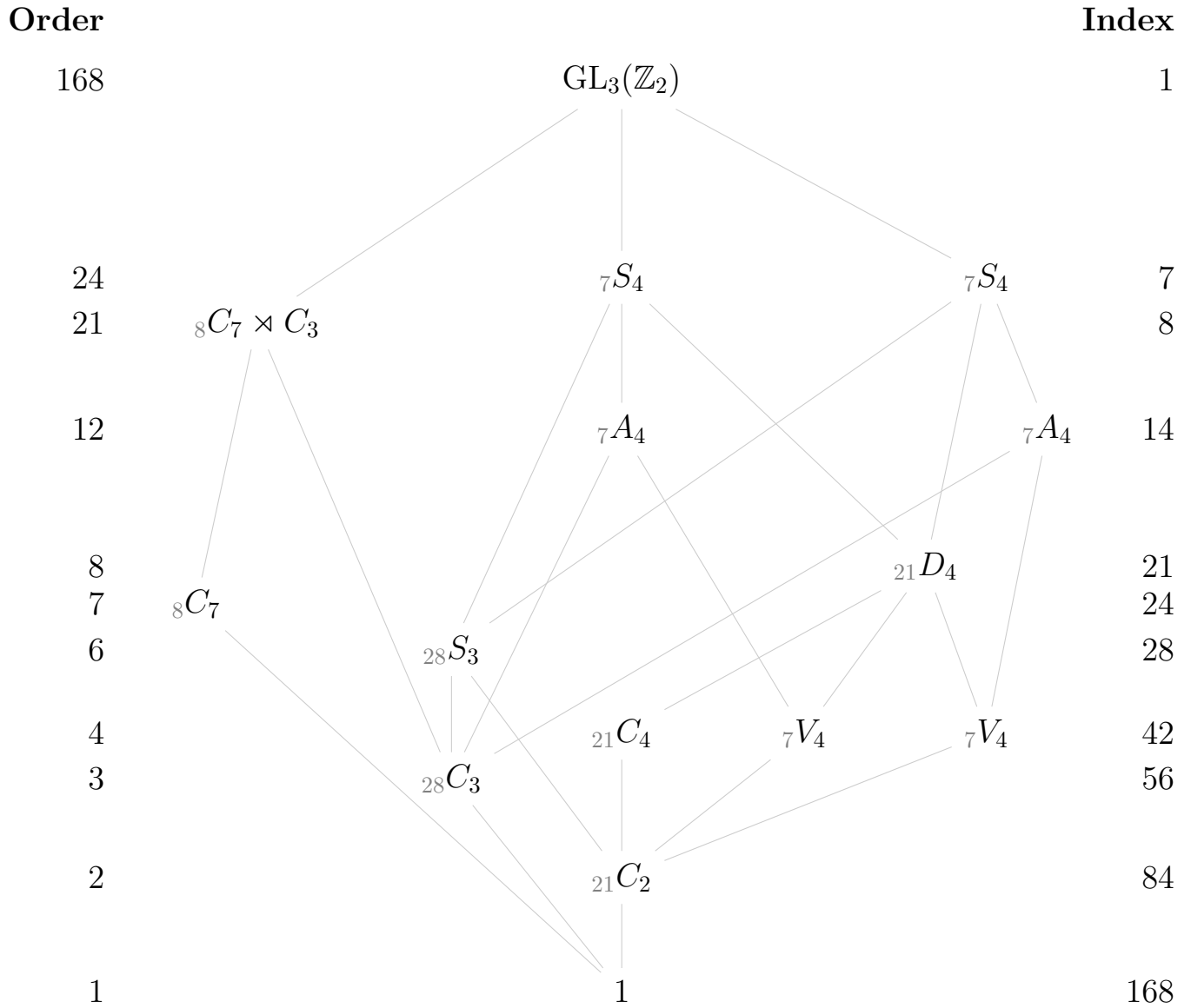


#1(c): A table of all 15 groups of order 24.

group	alias(es)	#subgroups	$n_2$	$P_2$	$n_3$	$P_3$
$C_{24}$	$C_8 \times C_3$					
$C_{12} \times C_2$						
$C_6 \times C_2^2$						
$S_4$						
$D_{12}$						
$\text{Dic}_{12}$						
$\text{SL}_2(\mathbb{Z}_3)$						
$C_3 \rtimes C_8$						
$C_3 \rtimes D_4$						
$A_4 \times C_2$						
$S_3 \times C_4$						
$D_4 \times C_3$						
$S_3 \times C_2^2$						
$Q_8 \times C_3$						
$\text{Dic}_6 \times C_2$						

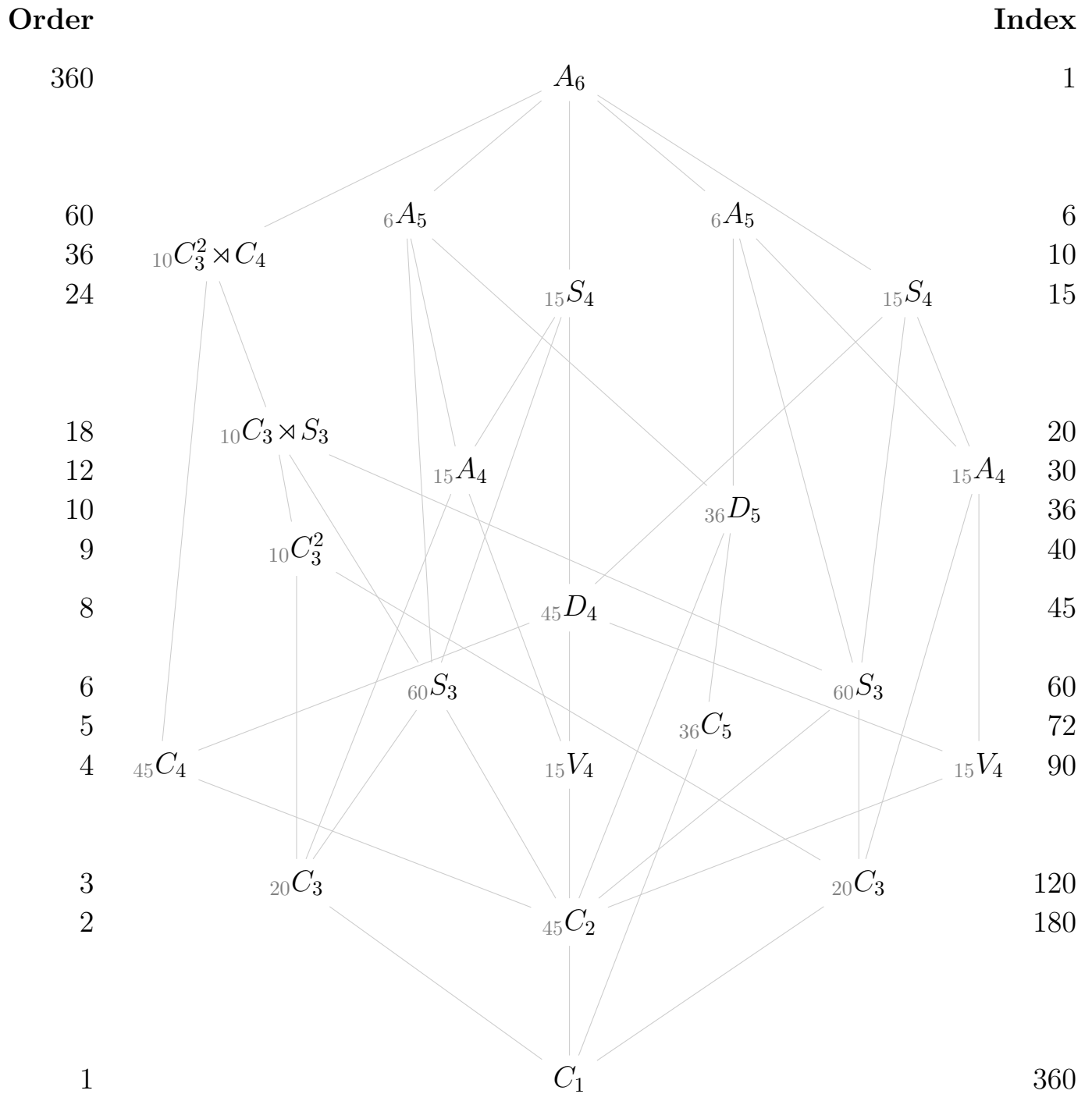
**#3(a)**: The subgroup lattice of the general linear group  $GL_3(\mathbb{Z}_2)$ , with:

- (i) the  $p$ -subgroups color-coded,
- (ii) arrows from each non-singleton conjugacy class  $cl(H)$  to the class  $cl(N(H))$  of its normalizer.



#5(a,b): The subgroup lattice of the alternating group  $A_6$ , with:

- (i) the  $p$ -subgroups color-coded,
- (ii) arrows from each non-singleton conjugacy class  $\text{cl}(H)$  to the class  $\text{cl}(N(H))$  of its normalizer.



**#5(c):** The ten subgroups of order  $90 = 2 \cdot 3^2 \cdot 5$ , the number of their subgroups, Sylow  $p$ -subgroups, and the isomorphism type of their Sylow 3-subgroup(s).

	#subgroups	$n_2$	$n_3$	$P_3$	$n_5$
$C_{90}$					
$C_{30} \times C_3$					
$D_{45}$					
$C_{15} \times D_3$					
$C_{15} \rtimes D_3$					
$C_9 \rtimes D_5$					
$C_3^2 \times D_5$					
$C_5 \rtimes D_9$					
$C_3 \times D_{15}$					
$C_3 \rtimes D_{15}$					